A taste of quantum computing: a gentle introduction using the IBM Quantum Computer.

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**Introduction**

IBM has put a quantum computer in the cloud that anyone can use for free. They've done this to encourage people to experiment. This set of notes provides a short and gentle introduction to both using this computer and to some of the basic ideas that underlie quantum computing. It is designed for the complete novice. No previous knowledge of anything to do with computing or quantum mechanics is required.

Being short, this introduction is far from being comprehensive. In fact, we will only look at three quantum gates, but, that said, these three gates are enough to illustrate some of the main differences between classical and quantum computing. *We see what it means for a qubit to be in a superposition of states and for two qubits to be entangled* — and what happens when we measure them.

These notes are designed to be used in conjunction with IBM’s Quantum Composer. We will work through twelve experiments. Each of which only takes about a minute to perform.
First steps with IBM’s Quantum Composer

The first step is to go the website https://quantumexperience.ng.bluemix.net/qx/experience and sign up for an account. You can do this by clicking the Sign In button on the top right. Accounts are free!

After you have made an account and signed in. You should get to a page that looks like the following:

![IBM Q Experience Welcome Page](image)

There are various links to guides on this page that are well worth exploring, but we will click the button to start experimenting.
You should now see a page that contains the following:

There are horizontal lines on the left. This is where we are going to construct what IBM calls our score (most computer scientists would call this a circuit). We will do this by dragging gates from the right and placing them on the lines on the left.

We start by clicking on the New button. Give the experiment a name and click on one of the buttons. (I usually click on ibmqx4.) We are now ready to start composing.

On the left of each of the horizontal lines (or wires) there is the symbol $|0\rangle$. This is a qubit. At the start of our experiment each of our five wires has the qubit $|0\rangle$ entered from the left. These then move from left to right. We manipulate these qubits by installing various gates that they have to pass through. The final step is to measure the qubits. When we measure a qubit we get a classical bit: either 0 or 1. When we measure $|0\rangle$, unsurprisingly, we get 0.

When we measure a qubit, the result is a classical bit.
We input qubits, but output classical bits.
Experiment 1. Measuring qubits.

Our first experiment will be to input the five qubits and the measure them. We will, of course, just get five 0s, but we will do it to see how we work with the composer and how the results are given.

On the right side underneath the gates there is a pink square with a meter drawn on it. This indicates that we are making a measurement. Drag one of these onto each of the lines. You should end up with something that looks like the picture below.

Notice that there are vertical arrows coming from each of the measuring devices. Each arrow has a number attached. These numbers refer to the ordering of the bits in the answer. Computer scientists always start counting with the number 0, not 1, so the five bits are labeled 0 through 4 — not 1 through 5. Also notice that the counting goes form right to left — not left to right. This will become clear as we continue. But the way that I have laid out the measuring devices is the best way. If you think of the inputs as starting at the top and moving down, the corresponding measurements start at the right and move left.

Quantum Circuit

Now that we have designed our score (circuit) we will run it. Here we have two choices. We can either run it on the actual quantum computer or we can run it on a simulated quantum computer. I suggest using the simulated computer for all of the examples in this manual.

Before pressing the simulate button, click on the settings button just to its right. Then click on edit parameters. You should get a window like the one below. We won’t ever change the seed from random, but we will change the number of shots. The number of shots is the number of times we run the circuit. For our example, once seems enough, but there is no harm in leaving the number of shots as 100. (I am not sure why IBM uses the word ‘shots.’ I am not a musician, but it looks to me as though they are mixing their metaphors — ‘recitals?’)
Now click on the simulate button. After running the simulator the results are displayed. I get:

![Bar chart showing result]

This is a bar chart showing the result. The 00000 at the bottom of the bar is the measurement. Each wire gave us a 0. (But remember the rightmost 0 corresponds to the topmost wire — the ordering of top-down gets translated to right-left.) The bar has height 1. If we multiply this by 100 we get a percentage, telling us that we got exactly the same result 100% of the time. In this case we ran the experiment 100 times and got 00000 every time.

Now that we have seen that measuring the qubit $|0\rangle$ gives us the classical bit 0, let’s start introducing gates so that we can perform some computation on the qubits before we measure them.
The X gate

The X gate changes the qubit $|0\rangle$ to the qubit $|1\rangle$. Unsurprisingly, when we measure the qubit $|1\rangle$, we get the classical bit 1.

**Experiment 2.**

The next experiment is to add an X gate to the top wire to give the circuit below.

When we measure the qubits we will get 1 for the top wire and 0s for the other wires. The result is below in which we see that 100% of the time (this is the 1.000 at the top of the bar) we get 00001, which is exactly what we expect.

The X gate flips the qubit $|1\rangle$ to the qubit $|0\rangle$. 

The X gate flips the qubits $|0\rangle$ and $|1\rangle$. 
$|0\rangle$ gets flipped to $|1\rangle$. 
$|1\rangle$ gets flipped to $|0\rangle$.
Experiment 3.

Our next experiment is to add a whole bunch of X gates. My circuit looks like the following.

The top wire has two of these gates, so \(|0\rangle\) gets flipped to \(|1\rangle\) and then back to \(|0\rangle\). When we measure this we will get 0. The second and third wires will give 1s after we measure the qubits. We expect to obtain 00110 after the measurements, and this is exactly what we get.
The Hadamard gate

The next gate we look at is called the Hadamard gate and is denoted by H. This gate sends the qubit $|0\rangle$ to $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ and it sends $|1\rangle$ to $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$. This needs to be explained, but we will put these two statements into a box so that we can refer back to them.

\[
\begin{align*}
\text{The Hadamard gate H sends} \\
|0\rangle & \text{ to } \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\
|1\rangle & \text{ to } \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle.
\end{align*}
\]

So far, we have only studied two qubits: $|0\rangle$ and $|1\rangle$. In fact, there are infinitely many qubits. In general, a qubit can have the form $a|0\rangle + b|1\rangle$ where $a$ and $b$ can be any two numbers with the property that the sum of their squares is 1. If both $a$ and $b$ are nonzero, then we call this mixture a superposition.

A qubit has the form $a|0\rangle + b|1\rangle$, where $a^2 + b^2 = 1$.

When you measure this qubit you get either 0 or 1.

The probability of getting 0 is $a^2$.

The probability of getting 1 is $b^2$.

We are now meeting some of the differences between classical and quantum computing. A qubit can be in a superposition of states — classical bits cannot. When we measure a qubit that is in a superposition of states we will get either 0 or 1 as an output. If we get 0, the qubit jumps to a new state of $|0\rangle$. If we get 1, the qubit jumps to a new state of $|1\rangle$. The fact that measurement causes the qubit to jump to a new state is something that belongs to quantum mechanics. It is not part of classical mechanics or classical computing.

One other thing that should be pointed out is that when we measure a qubit that it is in a superposition of states, we get 0 or 1 with given probabilities. There is inherent randomness.
When we measure the qubit \( \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \) we get 0 with probability \( \left( \frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2} \) and 1 with probability \( \left( \frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2} \). This means that both 0 and 1 are equiprobable.

When we measure the qubit \( \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \) we get 0 with probability \( \left( \frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2} \) and 1 with probability \( \left( \frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2} \). Once again, this means that both 0 and 1 are equiprobable.

We need to run a number of experiments to see what is going on.

**Experiment 4.**

For our next experiment we will just put one H gate on the top wire.

The qubit \( |0\rangle \) is input on the top wire. It then passes through the Hadamard gate and becomes \( \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \). When we measure this qubit we will get either 0 or 1. When we measure the other qubits we will just get 0s.

My results are:
Recall that we running the circuit 100 times. The results tell me that in 51 of the runs I got a 1 for the top qubit and for 49 of the runs I got a 0. This is like tossing a coin 100 times and getting heads 51 times and tails 49 times.

Of course, if I toss a coin 100 times and you toss a coin 100 times there is no reason that we should get exactly the same number of heads. The results for your experiment are probably different from mine. You will probably have roughly the same number of 0s as 1s for the top measurement. However, it is possible, but extremely unlikely, that you got 1s every time.

It is probably worth changing the number of shots to 1 and repeating this experiment several times to convince yourself that we get 0 and 1 at random and with equal probability whenever we measure the top qubit.

**Experiment 5.**

In the last experiment we sent \(|0\rangle\) through the Hadamard gate. We will now send \(|1\rangle\) through. The following circuit does this.
The X gate flips the qubit $|0\rangle$ to $|1\rangle$. This is then sent through the Hadamard gate where it becomes $\frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle$. When we measure this qubit we get 0 or 1, each with equal probability of $\frac{1}{2}$.

My results are:

**Quantum State: Computation Basis**

![Graph showing quantum state](chart.png)

The difference between $\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$ and $\frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle$.

As we have seen, when we measure $\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$ we get either 0 or 1 with equal probability. When we measure $\frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle$ we also get 0 or 1 with equal probability. They both give us exactly the same results. It leads us to ask if there is any difference between these two qubits. You might also be wondering whether the minus sign is important. After all, when we make a measurement, we square the number to get the probability: $\left(\frac{1}{\sqrt{2}}\right)^2 = \left(-\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$.

The answer is that the plus and minus signs are extremely important in quantum computing. They are also related to another question you might be wondering about: When we do computations we usually want a definite answer. We don’t want to get two different possible answers with equal probabilities. The Hadamard gate seems to introduce uncertainty, can it be used to reduce uncertainty?

The answer to this is Yes. The Hadamard gate can be used to reduce uncertainty in certain situations. To do this we also need to use both plus and minus signs. We illustrate this with two important examples.

First, we input $\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$ into a Hadamard gate. We know that $|0\rangle$ gets changed to $\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$, and $|1\rangle$ gets changed to $\frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle$. So, $\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$ will get
changed to \( \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right) + \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \right) \). If you expand and simplify this last expression you will see that it reduces to just \(|0\rangle\). The \(|1\rangle\)s cancel.

For the second example we input \( \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \) into the Hadamard gate. We get

\[
\frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right) - \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \right) .
\]

This simplifies to just \(|1\rangle\).

We will demonstrate these two examples with experiments 6 and 7. In experiment 6 we send \(|0\rangle\) through a Hadamard gate putting it into state \( \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \). We then send it through another Hadamard gate which puts it back into state \(|0\rangle\). Then we measure to check that we do, in fact, get 0 with certainty.

The circuit looks like:

My results are:

Exactly what we expect!
Experiment 7 looks like:

We start with $|0\rangle$ and flip it to $|1\rangle$ using the X gate. After going through the first Hadamard gate it is in state $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$. This is then sent through the second Hadamard gate and ends back in state $|1\rangle$. We then measure it to check that we do get 1.

My results are:

Again, this is exactly what we were expecting!

Algebraically, it is very simple to see that if we have two Hadamard gates next to one another on a wire, the second gate 'undoes' what the first gate does. The qubits entering the first gate will be identical to the qubits leaving the second gate. However, even though the algebra is simple, something very deep and important is going on. That we can introduce randomness using a quantum circuit is not really surprising. But what is surprising is that randomness can be eliminated. In experiment 6, if we were to measure after the top qubit after it had passed through the first Hadamard gate but before the second, we would get 0 or 1 completely at random. However, after the qubit has passed through the second Hadamard gate this uncertainty is entirely eliminated. Now we get 0 every time.
Experiments 8 and 9: Hadamard gates on more than one wire.

The next two experiments show what happens when we put H gates on more than one wire. It is useful to see the results of these experiments in order to compare them to the results we will see later after introducing the CNOT gate. Putting H gates on two or more wires does not entangle qubits. The CNOT gate does entangle them.

In the next experiment we place H gates on the first and second wires.

For both of these wires |0⟩ enters the H gate and is changed to \( \frac{1}{\sqrt{2}} |0⟩ + \frac{1}{\sqrt{2}} |1⟩ \). When we measure these qubits we will get either 0 or 1, each with equal probability. This is analogous to taking two coins and tossing them. Each coin is equally likely to land either heads or tails. If we list the outcome of the first coin as H or T and follow it with the outcome of the second coin, we will get one of HH, HT, TH or TT, each with equal probability, i.e., each with probability ¼. For our experiment, we expect to get 00, 01, 10 and 11 for the measurements on the first two wires. We should get each of these four outcomes randomly with probability ¼. (The other three wires will give 0s.)

My results give exactly what we predict:
The next experiment is to put H gates on the first three wires.

This is analogous to tossing three coins. For the top three wires, we expect to get 000, 001, 010, 011, 100, 101, 110 and 111 each with probability 1/8.

My results agree with this, but perhaps not convincingly. Remember that we are performing the experiment 100 times (100 shots). For this experiment it might have been better to have done it with a larger number. I suggest experimenting by varying the number of shots. The larger the number, the closer the proportions should be to 0.125.
Quantum State: Computation Basis
The CNOT gate

The gates we have looked at so far just acted on one qubit. The CNOT gate is not like this: it acts on two qubits. We will call these two qubits the control and target qubits. If the control bit is either $|0\rangle$ or $|1\rangle$, it is unchanged by the gate. If the control qubit is $|0\rangle$, then it leaves the target qubit as it is. If the control qubit is $|1\rangle$, then it acts like the X gate on the target qubit, i.e., it flips the $|0\rangle$s and $|1\rangle$s in the target qubit.

We will perform three experiments with the CNOT gate. The first two just illustrate its basic action. The third shows how it can be used to entangle two qubits — something that doesn’t occur in classical computing.

In the IBM Composer the second qubit is the control bit and the first qubit is the target bit. (This is a little unusual. It’s much more common to have the control bit being the first qubit and the target as the second. In practice, it doesn’t matter which order you choose, but if you look elsewhere you will probably find the CNOT gate with the dot [control] above the plus sign [target].)

In experiment 10, we set the top qubit (target) to $|0\rangle$ and the second qubit (control) to $|0\rangle$.

Since the control bit is $|0\rangle$, the target bit is unchanged and remains $|0\rangle$, so we expect the measurements to give us five $0$s. The results confirm this.
In experiment 11, we use an X gate to change the control bit to $|1\rangle$.

Since the control bit is $|1\rangle$, the target bit gets flipped and also becomes $|1\rangle$. We expect the measurements to give 00011, which the results confirm:

The next experiment is the really interesting one. We use the H gate to put the control bit into the superposition $\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$. As before we start with the target bit being $|0\rangle$. 
The results:

We can see that the measurements on the top two wires are either both 0 or both 1. The probabilities of getting two 0s is about equal to the probability of getting two 1s.

It is easy to misunderstand this result and think it uninteresting, but it is both interesting and important!

First, we will look at an argument that initially seems plausible, but is, in fact, completely wrong.

**Completely wrong argument:** The control bit is unchanged by the CNOT gate. The control bit is $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ before we measure it. We know that when we measure this qubit we will get 0 and 1 with equal probability. We also know that when the control bit is 0 the target bit is unchanged, but when the control bit is 1 the target bit gets flipped, so it is unsurprising that we either get two 0s or two 1s with equal probability.

There are a number of things wrong with this argument. The first sentence is false. It is true that if the control bit is either $|0\rangle$ or $|1\rangle$, then they are unchanged by the
CNOT gate, but, as we shall see, if the control bit is $\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$, then the gate does change it. Also, it is important to remember that we make the measurements after the qubits have passed through the gate. If the control bit remains as $\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$, what can the top qubit be changed into after passing through the gate? If it is of the form $a|0\rangle + b|1\rangle$, then when we measure, we will get 0 with probability $a^2$ and 1 with probability $b^2$, but there will be no correlation between these outcomes and the measurement of the control bit.

**Correct argument:** After passing through the CNOT gate the two qubits are in the entangled state $\frac{1}{\sqrt{2}} |0\rangle|0\rangle + \frac{1}{\sqrt{2}} |1\rangle|1\rangle$. When we measure these qubits we will get either 00 or 11 with equal probability. It is called an entangled state because we cannot describe the two qubits as having two separate states. We cannot talk of the state of the top qubit or the state of the bottom qubit, only the state that describes both qubits.

Suppose that we have two qubits in the entangled state $\frac{1}{\sqrt{2}} |0\rangle|0\rangle + \frac{1}{\sqrt{2}} |1\rangle|1\rangle$ and that we decide to make a measurement on one of them, the measurement will give 0 and 1, with equal probability. As soon as the measurement is made, the entangled state disentangles. If the measurement is 0, then both qubits jump to state $|0\rangle$. If the measurement is 1, then both qubits jump to state $|1\rangle$. Consequently, when the second measurement is made, it gives the same result as the first measurement.

Two qubits in the entangled state $\frac{1}{\sqrt{2}} |0\rangle|0\rangle + \frac{1}{\sqrt{2}} |1\rangle|1\rangle$ have many uses. We haven’t talked about how qubits can be physically realized, but one way is using the polarization of photons. Two photons sharing an entangled state don’t have to be near one another but can be miles apart. When the polarization of the photons is measured they will agree, both giving 0 or both giving a 1. (This is what Einstein meant when he talked about *spooky action at a distance.*) Photons in this state are important for quantum key distribution systems and for quantum teleportation. Many experiments have been performed using photons in entangled states.

**Final comment**

This has just been a taste of quantum computing. If your appetite has been whetted, then I encourage you to play around constructing other circuits and exploring the other gates. Some helpful information is posted on the IBM website. There are many amazing results that need only a little more mathematics to understand. Many introductory books explain things more fully. My suggestion — though I might be biased — is to read *Quantum Computing for Everyone*. 