# 125 HW #11

4.5: 14

4.7: 2, 33a

4.8: 8, 11

**4.5 #14**

$$f(x) = \frac{x^2}{x^2+9} \qquad f'(x) = \frac{(x^2+9)\cdot 2x - x^2\cdot(2x)}{(x^2+9)^2}$$

$$= \frac{18x}{(x^2+9)^2}$$

$$f''(x) = \frac{(x^2+9)^2\cdot 18 - 18x\cdot 2(x^2+9)\cdot 2x}{(x^2+9)^4}$$

$$= \frac{18(x^4+18x^2+81) - 18(4x^4+36x^2)}{(x^2+9)^4}$$

$$= 18\,\frac{-3x^4-18x^2+81}{(x^2+9)^4}$$

$$= -54\,\frac{x^4+6x^2-27}{(x^2+9)^4} = -54\,\frac{(x^2+9)(x^2-3)}{(x^2+9)^4}$$

$$= -54\,\frac{x^2-3}{(x^2+9)^4} \qquad \text{finally.}$$

Asymptotes: vert: none (denom never 0)

horz: $\lim_{x\to\infty} f(x) = 1$, $\lim_{x\to-\infty} f(x) = 1$

so $y=1$ is a horz. tote.

Crit pts: $f'(x) = 0$ means numerator $= 0$

so $x = 0$

$f''$: $f''(x) = 0$ means $x^2-3=0$

$x = \pm\sqrt{3}$.

| | $-\sqrt{3}$ | $0$ | $\sqrt{3}$ |
|---|---|---|---|
| $f$: | $3/18 = 1/6$ | $0$ | $1/6$ |

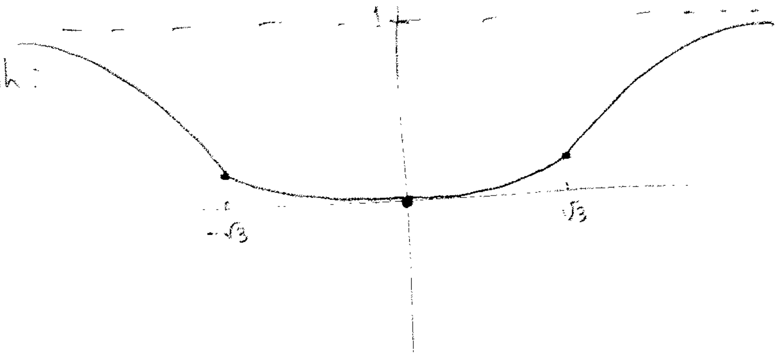
$f'$: $- - - - -$ $0$ $+ + + + +$

$f''$: $- - -$ $0$ $+ + + + + +$ $0$ $- - -$

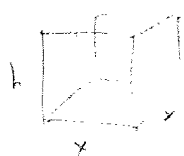
intercepts: $y$: $f(0) = 0$

$f(x) = 0$ means $x^2 = 0$ so $x = 0$ is the only $x$-intercept

Graph:




## 4.7 #12




$V = 32000 \text{ cm}^3$.

no top, minimize the surf. area.

$A = x^2 + 4xh$, $\quad V = x^2 h = 32000$

so $h = V/x^2 = \dfrac{32000}{x^2}$

$A(x) = x^2 + 4x \cdot 32000/x^2 = x^2 + 128000\, x^{-1}$

$A'(x) = 2x - 128000\, x^{-2}$

$A'(x) = 0 \rightarrow 2x - 128000\, x^{-2} = 0$

$2x^3 - 128000 = 0$

$x^3 = 64000$

$x = 40$

So the minimal dimensions are: $x = 40$,

$h = \dfrac{32000}{x^2} = 20$

4.7 # 33


total length is $4x+3y$, which equals 10.

need to ~~min~~/maximize the Area.

$$A = x^2 + \frac{1}{2}\cdot y \cdot \frac{\sqrt{3}}{2}y = x^2 + \frac{\sqrt{3}}{4}y^2.$$

$$4x+3y=10 \qquad y = \frac{10-4x}{3}$$

so $$A = x^2 + \frac{\sqrt{3}}{4}\left(\frac{10}{3}-\frac{4}{3}x\right)^2$$

$$A'(x) = 2x + \frac{\sqrt{3}}{\cancel{4}}\cdot 2\left(\frac{10}{3}-\frac{4}{3}x\right)\cdot -\frac{4}{3}$$

$$= 2x - \frac{2\sqrt{3}}{3}\left(10/3 - 4/3\,x\right) = 2x - \frac{20\sqrt{3}}{9} + \frac{8\sqrt{3}}{9}x = 3.539x - 3.849$$

$A'(x) = 0$ for $\quad 2x + \frac{8\sqrt{3}}{9}x = \frac{20\sqrt{3}}{9}$

$$(18+8\sqrt{3})x = 20\sqrt{3}$$

$$x = 1.087 = \frac{40\sqrt{3}}{9+4\sqrt{3}}$$

Possible values for x are 0 to 2.5.

need abs extrema for $A(x)$

| $x$ | $A(x)$ |
|---|---|
| 0 | $\frac{10^2\sqrt{3}}{4\cdot 3^2} = 4.81$ |
| 2.5 | $2.5^2 = 6.25$ |
| $\frac{40\sqrt{3}}{9+4\sqrt{3}}$ | $0 = 2.718$ |

min is when $x = 1.087$

max is when $x = 2.5$

(all square no triangle)

**4.8 #8** $f(x) = x^5 + 2$ $\quad x_1 = -1$ $\quad f'(x) = 5x^4$

$$x_2 = -1 - \frac{f(-1)}{f'(-1)}$$

$$\frac{f(x)}{f'(x)} = \frac{x^5+2}{5x^4} = \frac{1}{5}x + \frac{2}{5}x^{-4}$$

$$= -1.2$$

$$x_3 = -1.2 - \frac{f(-1.2)}{f'(1.2)} = -1.152$$

**4.8 #11** $\sqrt[5]{20} \longrightarrow f(x) = x^5 - 20$ $\quad f'(x) = 5x^4$

we'll start with $x_1 = 2$

$$x_2 = 2 - \frac{f(2)}{f'(2)} = 1.85$$

$x_3 = 1.82148614$

$x_4 = 1.82056514$

$x_5 = 1.82056420$

$\sqrt[5]{20} \approx 1.82056420$