271 HW #10

15.4 4, 17

15.5 5, 13, 35

15.4 #4 $z = y \ln x$ at $(1,4,0)$

tgt plane: $z - 0 = \frac{\partial z}{\partial x}(1,4,0)(x-1) + \frac{\partial z}{\partial y}(1,4,0)(y-4)$

$\frac{\partial z}{\partial x} = \frac{y}{x}$ at $(1,4)$: 4

$\frac{\partial z}{\partial y} = \ln x$ at $(1,4)$: $\ln 1 = 0$

$\rightarrow z = 4(x-1) + 0$

$z = 4x - 4.$

15.4 #17 $z = \frac{2x+3}{4y+1}$

$z_0 = \frac{3}{1} = 3$

$\frac{\partial z}{\partial x} = \frac{2}{4y+1}$ $(0,0) \rightarrow 2$

$\frac{\partial z}{\partial y} = -(2x+3)(4y+1)^{-2} \cdot 4$

$(0,0) \rightarrow -(3)(1)^{-2} \cdot 4 = -12$

tgt plane: $z = 3 + 2x - 12y$

so $\frac{2x+3}{4x+1} \approx 3 + 2x - 12y$ near $(0,0)$.

15.5 #5 $w = x e^{y/z}$ $x = t^2$ $y = 1 - t$ $z = 1 + 2t$

$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$

$= e^{y/z} \cdot 2t + \frac{x}{z} e^{y/z} \cdot (-1) + x e^{y/z} \cdot \frac{-1}{z^2} \cdot 2$

$= e^{y/z}\left(2t - \frac{x}{z} - \frac{2xy}{z^2}\right)$

15.5 #13

$z = f(x,y) \qquad x = g(t) \qquad y = h(t)$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$t = 3 \rightarrow x = g(3) = 2$

$y = h(3) = 7$

$$= f_x(2,7) \cdot g'(3) + f_y(2,7) \cdot h'(3)$$

$$= 6 \cdot 5 + -8 \cdot -4$$

$$= 30 + +32 = 62.$$

15.5 #35

$T_x(2,3) = 4$

$T_y(2,3) = 3$

$x = \sqrt{1+t}$ $\qquad \frac{dx}{dt} = \frac{1}{2}(1+t)^{-1/2}$

$y = 2 + \frac{1}{3}t$ $\qquad \frac{dy}{dt} = \frac{1}{3}$

Want $\frac{dT}{dt}$ at $t = 3$.

$x(3) = 2$

$y(3) = 3$

$$\frac{dT}{dt} = T_x(2,3) \cdot \frac{dx}{dt}(3) + T_y(2,3)\frac{dy}{dt}(3)$$

$$= 4 \cdot \frac{1}{2}(4)^{-1/2} + 3 \cdot \frac{1}{3}$$

$$= 1 + 1 = \underline{2}$$