271 HW # 11

15.5 24, 29, 36

15.6 8, 16

15.5 #24 $M = xe^{y-z^2}$ $x = 2uv$, $y = u - v$, $z = u + v$

$$\frac{\partial M}{\partial u} = \frac{\partial M}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial M}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial M}{\partial z}\frac{\partial z}{\partial u}$$

$$= e^{y-z^2} \cdot 2v + xe^{y-z^2} \cdot 1 + -2zxe^{y-z^2} \cdot 1$$

at $u = 3$ $v = -1$, $x = -6$ $y = 4$ $z = 2$

$$= e^0 \cdot 2 \cdot -1 + -6e^0 \cdot 1 - 2 \cdot 2 \cdot -6 \cdot e^0 \cdot 1$$

$$= -2 - 6 + 24 = 16$$

15.5 #29 $\cos(x-y) = xe^y$

$$F(x,y) = xe^y - \cos(x-y)$$

$F_x = e^y + \sin(x-y)$

$F_y = xe^y - \sin(x-y)$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{e^y + \sin(x-y)}{xe^y - \sin(x-y)}.$$

#36 a) $\frac{\partial W}{\partial T} = -2$, so W decreases as Temp increases (keeping R same)

$\frac{\partial W}{\partial R} = 8$ so W increases as Rain increases (keeping T same)

b) $$\frac{dW}{dt} = \frac{\partial W}{\partial R} \cdot \frac{dR}{dt} + \frac{\partial W}{\partial T}\frac{dT}{dt}$$

$$= 8 \cdot .1 + -2 \cdot .15 = .8 - .3 = .1$$

15.6 #8

$f(x,y) = y^2/x$ $\quad P=(1,2)$ $\quad \vec{u} = \langle \frac{2}{3}, \frac{\sqrt{5}}{3} \rangle$

a). $\nabla f = \langle -\frac{y^2}{x^2}, 2y/x \rangle$

b) $\nabla f(1,2) = \langle$ [illegible] $\rangle \langle -4, 4 \rangle$

$=$ [illegible]

c) $D_{\vec{u}} f(P) = \nabla f \cdot \vec{u}$

$= \langle -4, 4 \rangle \cdot \langle 2/3, \sqrt{5}/3 \rangle$

$= -8/3 + 4\sqrt{5}/3$

15.6 # 16 $\quad f(x,y,z) = (xyz)^{1/2}$ at $(3,2,6)$ $\quad \vec{v} = \langle -1, -2, 2 \rangle$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\vec{v}}{\sqrt{1+4+4}} = \langle -1/3, -2/3, 2/3 \rangle$$

$D_{\vec{u}} f(3,2,6) = \nabla f(3,2,6) \cdot \vec{u}$

$= \langle \frac{1}{2}(xyz)^{-1/2} \cdot yz, \frac{1}{2}(xyz)^{-1/2} \cdot xz, \frac{1}{2}(xyz)^{-1/2} \cdot xy \rangle (3,6,2) \cdot \vec{u}$

$= \langle \frac{12}{12}, \frac{6}{12}, \frac{18}{12} \rangle \cdot \langle -1/3, -2/3, 2/3 \rangle$

$= \langle 1, 1/2, 3/2 \rangle \cdot \langle -1/3, -2/3, 2/3 \rangle$

$= -1/3 + -1/3 + 1 = 1/3.$