271 HW #12

15.6 21, 30, 40a

15.7 5, 8

<u>15.6 #21</u> $f(x,y) = y^2/x = x^{-1}y^2$ at $(2,4)$

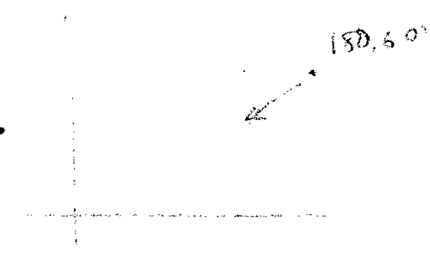
The direction is $\nabla f(2,4) = \langle -x^{-2}y^2,\ 2x^{-1}y \rangle$

$$\begin{matrix} x=2 \\ y=4 \end{matrix} \longrightarrow \left\langle -\frac{16}{4},\ 2\cdot\frac{1}{2}\cdot 4 \right\rangle$$

$$= \langle -4, 4 \rangle$$

The rate of change is $|\nabla f| = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

<u>15.6 #30</u> $z = 200 + .02x^2 - .001y^3$



The direction of travel is $\langle -80, -60 \rangle$

$\vec{u} = \langle -80, -60 \rangle$

$$D_{\vec{u}} = \nabla f(80,60) \cdot \vec{u}$$

$$\nabla f = \langle .04x,\ -.003y^2 \rangle$$

$$\begin{matrix} x=80 \\ y=60 \end{matrix} \quad = \langle 3.2, -10.8 \rangle$$

$$D_{\vec{u}} = \langle 3.2, -10.8 \rangle \cdot \langle -80, -60 \rangle$$

$$= 392$$

negative, so z is <u>increasing</u>

so its getting <u>deeper</u>.

Technically $\vec{u}$ should be a unit vector, but we only care about the sign of $D_{\vec{u}}$, so it doesn't matter

15.6 #40a $\quad y = x^2 - z^2 \quad$ tgt plane at $(4,7,3)$

it's a level curve for $f(x,y,z) = x^2 - y - z^2$

The normal vector is $\nabla f\ (4,7,3)$

$$\nabla f\ (4,7,3) = \langle 2x, -1, -2z \rangle$$

at $(4,7,3) \longrightarrow \langle 8, -1, -6 \rangle$

this is the normal vec, so the plane is

$$8(x-4) - 1(y-7) - 6(z-3) = 0$$

15.7 # 5 $\quad f(x,y) = 9 - 2x + 4y - x^2 - 4y^2$

crit pts: $\quad f_x = -2 - 2x \qquad -2-2x=0 \rightarrow x=-1$

$\qquad f_y = 4 - 8y \qquad 4-8y=0 \rightarrow y = \frac{1}{2}$

so $(-1, \frac{1}{2})$ is a crit pt.

$$f_{xx} = -2 \qquad f_{xy} = 0 \qquad f_{yy} = -8$$

$$D = f_{xx} f_{yy} - f_{xy}^2 = 16 > 0$$

$f_{xx} < 0$ so $(-1, \frac{1}{2})$ is a local max.

15, 7 #8

$$f(x,y) = e^{4y-x^2-y^2}$$

crit pts: $f_x = e^{4y-x^2-y^2} \cdot -2x \quad \rightarrow \quad -2x = 0 \qquad x=0$

$f_y = e^{4y-x^2-y^2} \cdot (4-2y) \quad \rightarrow \quad 4-2y=0 \qquad y=2$

(0,2) is the crit pt.

$$f_{xx} = e^{4y-x^2-y^2} \cdot -2 + -2x \cdot e^{4y-x^2-y^2} \cdot -2x$$

$$f_{yy} = e^{4y-x^2-y^2} \cdot -2 + (4-2y)\, e^{4y-x^2-y^2} \cdot (4-2y)$$

$$f_{xy} = -2xe^{4y-x^2-y^2} \cdot (4-2y)$$

at (0,2) : $f_{xx} = -2e^4 \qquad f_{xy} = 0$

$f_{yy} = -2e^4$

so $D = f_{xx} f_{yy} - f_{xy}^2 = 4e^8 > 0$

and $f_{xx} < 0$ so (0,2) is a <u>max</u>