271 HW #9

15.2 10, 31

15.3 6, 28, 61

15.2 #10 $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2}$.

Approaching along $x=0$ (y-axis) we get

$$\lim_{y\to 0} \frac{\sin^2 y}{y^2} = \lim_{y\to 0}\left(\frac{\sin y}{y}\right)^2 = 1 \quad \text{(by L'Hôpital)}$$

Approaching along $y=0$ (x-axis) we get

$$\lim_{x\to 0} \frac{x^2}{2x^2} = \frac{1}{2}$$

Limits are different, so the original lim DNE.

15.2 #31 Where is $F(x,y) = \arctan(x+\sqrt{y})$ continuous?

arctan is cont. everywhere,

so $F(x,y)$ is cont. whenever $x+\sqrt{y}$ is,

which is when $y \geq 0$.

15.3 #6 $f_x(-1,2)$ is ~~pos.~~ neg.

$f_y(-1,2)$ is neg.

15.3 #28

$$f(x,y) = \int_y^x \cos(t^2)\,dt$$

$$f_x = \frac{\partial}{\partial x}\int_y^x \cos(t^2)\,dt = \cos(x^2)$$

$$f_y = \frac{\partial}{\partial y}\int_y^x \cos(t^2)\,dt = -\frac{\partial}{\partial y}\int_x^y \cos t^2\,dt = -\cos(y^2)$$

15.3 #61

$$f(x,y) = 3xy^4 + x^3y^2$$

$$f_x = 3y^4 + 3x^2y^2 \qquad f_y = 12xy^3 + 2x^3y$$

$$f_{xx} = 6xy^2 \qquad f_{yy} = 36xy^2 + 2x^3$$

$$\boxed{f_{xxy} = 12xy \qquad f_{yyy} = 72xy}$$