126 HW 1

5.2 # 5c, 21, 23, 34, 43

5c $\Delta x = 2$, it's

$$\int_0^8 f(x)\,dx \approx 2\left(f(1) + f(3) + f(5) + f(7)\right)$$

$$= 2(3 + 2 + 1 + -1) = 10$$

21 $\displaystyle\int_{-1}^{5} 1 + 3x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$ $\qquad \Delta x = \frac{5 - -1}{n} = \frac{6}{n}$

$x_i = -1 + \frac{6i}{n}$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + 3\left(-1 + \frac{6i}{n}\right)\right) \cdot \frac{6}{n}$$

$$= \lim_{n\to\infty} \frac{6}{n} \sum_{i=1}^{n} 1 - 3 + \frac{18i}{n} = \lim_{n\to\infty} \frac{6}{n} \sum_{i=1}^{n} -2 + \frac{18i}{n}$$

$$= \lim_{n\to\infty} \frac{6}{n}\left(-2n + \frac{18}{n}\,\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty} -12 + \frac{18 \cdot 6}{2n^2} \cdot n(n+1) = -12 + 9 \cdot 6$$

$$= 42$$

23 $\displaystyle\int_0^2 2 - x^3\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$ $\qquad \Delta x = \frac{2}{n}$ $\qquad x_i = \frac{2i}{n}$

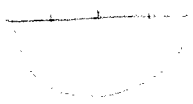
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(2 - \left(\frac{2i}{n}\right)^3\right) \cdot \frac{2}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} - \frac{8i^3}{n^5}$$

$$= \lim_{n\to\infty} \frac{4}{n} \cdot n - \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) = 4 - \frac{8 \cdot 2}{6}$$

$$= 4 - 8/3 = 4/3$$

34 a. 

$$\int_0^2 g(x)\,dx = \tfrac{1}{2}\cdot 2\cdot 4 = 4$$

b. 

$\int_2^6 g(x)\,dx$ is = half circle, radius 2, negative

$$= -\tfrac{1}{2}\pi\cdot 2^2 = -2\pi$$

from 6 to 7 it's $\tfrac{1}{2}$,

c. so $\int_0^7 g(x)\,dx = 4 - 2\pi + \tfrac{1}{2} = 4.5 - 2\pi$

43 $\int_0^1 5 - 6x^2\,dx = 5\int_0^1 dx - 6\int_0^1 x^2\,dx$

$= 5\cdot 1 - 6\cdot\tfrac{1}{3} = 5 - 2 = 3.$