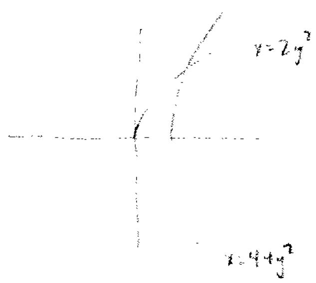
126 HW #4

6.1 #19

6.2 2, 7, 22

6.3 2

6.1 #19 $x = 2y^2, \quad x = 4 + y^2$ area between.



intersect at: $2y^2 = 4 + y^2$

$y^2 = 4$

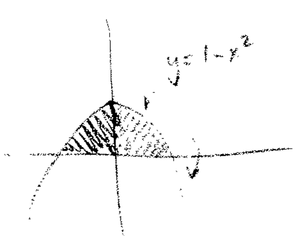
$y = \pm 2.$

$$\int_{-2}^{2} 4 + y^2 - 2y^2 \, dy = \int_{-2}^{2} 4 - y^2 \, dy = 4y - y^3/3 \Big|_{-2}^{2}$$

$$= 8 - \frac{8}{3} - \left(-8 + \frac{8}{3}\right) = 16 - \frac{16}{3} = \frac{32}{3} = 10.6\bar{6}$$

6.2 #2

slices are circles, $A(x) = \pi (1 - x^2)^2$

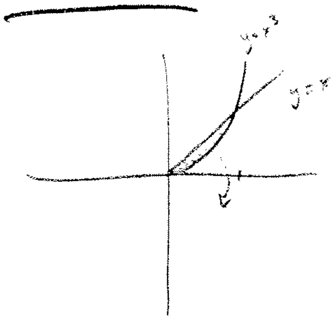


$$V = \int_{-1}^{1} \pi (1 - x^2)^2 \, dx = \pi \int_{-1}^{1} 1 - 2x^2 + x^4 \, dx$$

$$= \pi \left(x - \frac{2}{3}x^3 + \frac{x^5}{5} \right)\Big|_{-1}^{1} = \pi \left[\left(1 - \frac{2}{3} + \frac{1}{5}\right) - \left(-1 + \frac{2}{3} - \frac{1}{5}\right) \right] = \frac{16}{15}\pi = 3.35$$

$$= 1.0\bar{6}\pi$$

6.2 #7



Slices are annuli,

outer radius $= x$

inner $= x^3$

so $A(x) = \pi\left(-(x^3)^2 + x^2\right)$

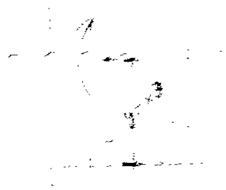
$= \pi\left(-x^6 + x^2\right)$

$$V = \int_0^1 \pi(-x^6 + x^2)\,dx = \pi \left(\frac{x^7}{7} + \frac{x^3}{3}\right)\Big|_0^1 = \pi\left(-\frac{1}{7} + \frac{1}{3}\right)$$

$$= \frac{4\pi}{21} = .19\,\pi$$

$$= .598$$

6.2 #22



Slices are annuli. inner radius $1-x^3$

outer 1

So $A(x) = \pi\left(1 - (1-x^3)^2\right) = \pi\left(1 - (1 - 2x^3 + x^6)\right)$

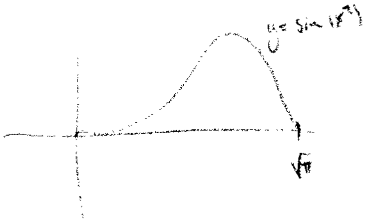
$= \pi\left(2x^3 - x^6\right)$

$$V = \int_0^1 \pi(2x^3 - x^6)\,dx = \pi\left(\frac{2x^4}{4} - \frac{x^7}{7}\right)\Big|_0^1 = \pi\left(\frac{1}{2} - \frac{1}{7}\right)$$

$$= \pi \cdot \frac{5}{14}$$

6.3 #2

$$V = \int_0^{\sqrt{\pi}} 2\pi x \cdot \sin(x^2)\,dx \qquad u = x^2, \quad du = 2x\,dx$$



$$= \pi \int_0^{\sqrt{\pi}} \sin u\,du = -\pi \cos u \Big|_{x=0}^{\sqrt{\pi}}$$

$$= -\pi \cos x^2 \Big|_0^{\sqrt{\pi}} = -\pi\left(\cos\pi - \cos 0\right)$$

$$= -\pi(-1 - 1) = 2\pi$$