

Math 126 HW #6

6.4 # 5, 8, 10, 13, 24

6.4 #5

It's the area: $60 + 120 = 180 \text{ J}$

6.4 #8

$$25 = k \cdot 10 \quad k = 2.50$$

$$\text{So } W = \int_0^{0.05} 2.50x \, dx = 1.25x^2 \Big|_0^{0.05} = 1.25 \cdot (0.05)^2 = .3125 \text{ J}$$

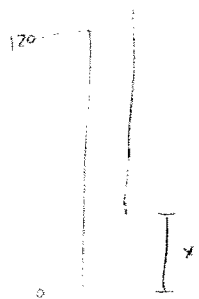
$= 5/16$

~~$W = \int_0^{0.05} 12x \, dx = 6x^2 \Big|_0^{0.05} = 6 \cdot \frac{9}{16} = \frac{27}{8} \text{ ft} \cdot \text{lb}$~~

6.4 #13a

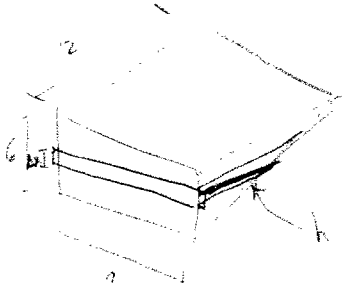
$$\text{weight}(x) = .5 \cdot (120 - x)$$

50 ft rope, x starts at 70.



$$\begin{aligned} \text{a) } W &= \int_{70}^{120} \frac{1}{2} (120 - x) \, dx = \int_{70}^{120} 60 - \frac{x}{2} \, dx \\ &= 60x - \frac{x^2}{4} \Big|_{70}^{120} = 60(120 - 70) - \frac{1}{4}(120^2 - 70^2) \\ &= 60 \cdot 50 - \frac{1}{4}(9500) \\ &= 625 \text{ ft} \cdot \text{lb} \end{aligned}$$

6.4 # 24.

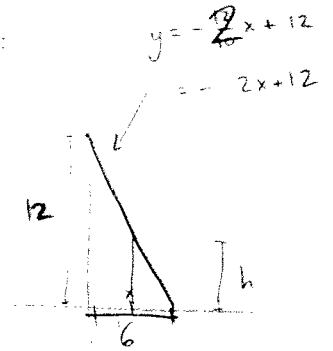


a slice of thickness Δx weighs:

$$\Delta x \cdot 625 \cdot 10 \cdot h$$

h is given by:

$$h = -2x + 12$$



so weight of the slice is

$$\Delta x \cdot 625 \cdot 10 \cdot (-2x + 12)$$

pump it distance x .

$$W \approx \sum x \cdot 625 \cdot 10 \cdot (-2x + 12) \Delta x$$

$$W = \int_0^6 x \cdot 625 \cdot 10 \cdot (-2x + 12) dx$$

$$= 625 \int_0^6 -2x^2 + 12x dx$$

$$= 625 \left(-\frac{2}{3}x^3 + 6x^2 \right) \Big|_0^6$$

$$= 625 \left(-\frac{2}{3}6^3 + 6 \cdot 6^2 \right)$$

$$= 625 \left(\frac{1}{3} \cdot 6^3 \right) = 625 \cdot 72 = 45000 \text{ ft lb}$$

6.4 # 10

for 1 ft:

$$12 = \int_0^1 kx dx = k \frac{x^2}{2} \Big|_0^1 = k/2$$

$$\text{so } k = 24$$

for $9/4$ ft:

$$W = \int_0^{9/4} 24x dx = 12x^2 \Big|_0^{9/4} = 12 \frac{9}{16} = \frac{27}{4} = 6.75 \text{ ft lb}$$