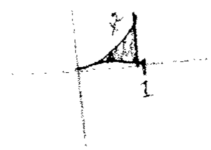
272 HW #2

16.3 13, 40, 52

16.4 10, 23

16.3 #13

$$\iint_D x\cos y \, dA$$



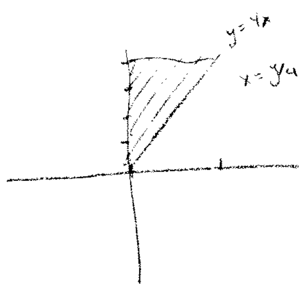
$$\int_0^1 \int_0^{x^2} x\cos y \; dy\, dx = \int_0^1 x \cdot \sin y \Big|_0^{x^2} dx$$

$$= \int_0^1 x \sin(x^2)\, dx = -\frac{1}{2}\cos x^2 \Big|_0^1 = -\frac{1}{2}\cos 1 - \left(-\frac{1}{2}\cos 0\right)$$

$$= -\frac{1}{2}\cos 1 + \frac{1}{2}.$$

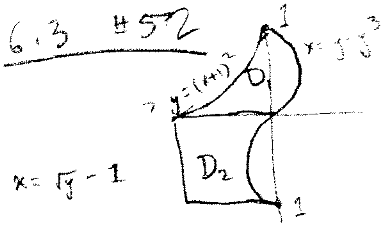
16.3 #40

$$\int_0^1 \int_{4x}^4 f(x,y)\, dy\, dx \qquad \text{change the order:}$$



$$\| \qquad \int_0^4 \int_0^{y/4} f(x,y)\, dx\, dy$$

16.3 #52



$$\iint_D y\, dA = \iint_{D_1} y\, dA + \iint_{D_2} y\, dA$$

$$= \int_0^1 \int_{\sqrt{y}-1}^{y-y^3} y\, dx\, dy + \int_{-1}^0 \int_{-1}^{y-y^3} y\, dx\, dy$$

$$= \int_0^1 y(y-y^3) - y(\sqrt{y}-1)\, dy + \int_{-1}^0 y(y-y^3) + y\, dy$$

$$= \int_0^1 y^2 - y^4 - y^{3/2} + y\, dy + \int_{-1}^0 y^2 - y^4 + y\, dy$$

$$= \frac{y^3}{3} - \frac{y^5}{5} - \frac{2}{5}y^{5/2} + \frac{y^2}{2}\Big|_0^1 + \frac{y^3}{3} - \frac{y^5}{5} + \frac{y^2}{2}\Big|_{-1}^0$$

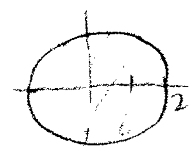
$$= \frac{1}{3} - \frac{1}{5} - \frac{2}{5} + \frac{1}{2} + -\left(-\frac{1}{3} + \frac{1}{5} + \frac{1}{2}\right)$$

$$= \frac{1}{3} - \frac{3}{5} + \frac{1}{2} + \frac{1}{3} - \frac{1}{5} - \frac{1}{2} = \frac{2}{3} - \frac{4}{5} = -2/15$$

16.4 #10

$$\iint_R \sqrt{4-x^2-y^2}\, dA$$

R :



$$= \int_0^2 \int_{-\pi/2}^{\pi/2} \sqrt{4-r^2}\; r\, d\theta\, dr$$

$$= \int_0^2 \pi \cdot \sqrt{4-r^2}\; r\, dr\, d\theta = -\pi \frac{1}{2} \frac{2}{3}(4-r^2)^{3/2}\Big|_0^2$$

$$= -\frac{\pi}{3}(0 - 4^{3/2}) = \frac{\pi}{3} \cdot 2^3 = \frac{8\pi}{3}$$

16.4 #23

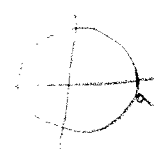
Vol of sphere, rad a

sphere is $x^2+y^2+z^2 = a^2$

i.e. $z = \sqrt{a^2-x^2-y^2}$

$= \sqrt{a^2-r^2}$

$$V = 2\iint_R \sqrt{a^2-r^2}\, dA$$



$$= 2\int_0^a \int_0^{2\pi} \sqrt{a^2-r^2}\; r\, d\theta\, dr.$$

= as above

$$= 2\left(-\frac{2\pi}{3}\left(0 - (a^2)^{3/2}\right)\right) = 2 \cdot \frac{2\pi}{3} \cdot a^3 = \frac{4}{3}\pi a^3$$