Math 272 HW #3

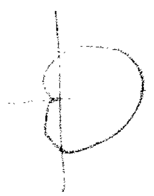
16.4 # 16

16.5 # 3, 5

16.6 # 2, 5

16.4 #16

$r = 4 + 3\cos\theta$



$$\iint_D r\,dr\,d\theta = \int_0^{2\pi}\int_0^{4+3\cos\theta} r\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{r^2}{2}\Big|_0^{4+3\cos\theta} d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} (4+3\cos\theta)^2\,d\theta$$

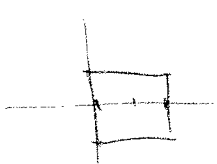
$$= \frac{1}{2}\int_0^{2\pi} 16 + 24\cos\theta + 9\cos^2\theta\,d\theta$$

$$= \frac{1}{2}\left(16\theta + 24\sin\theta + 9\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\right)\Big|_0^{2\pi}$$

$$= \frac{1}{2}\left(32\pi + \frac{9}{2}\cdot 2\pi\right) = \frac{1}{2}\cdot 41\pi = \frac{41\pi}{2}$$

16.5 #3

$\rho(x,y) = xy^2$



$$m = \int_0^2\int_{-1}^1 xy^2\,dy\,dx = \int_0^2 x\,dx \int_{-1}^1 y^2\,dy$$

$$= \frac{x^2}{2}\Big|_0^2 \cdot \frac{y^3}{3}\Big|_{-1}^1$$

$$= 2 \cdot \frac{2}{3} = 4/3.$$

$$\bar{x} = \frac{1}{m}\iint x^2y^2 = \frac{3}{4}\left(\int_0^2 x^2\,dx\right)\left(\int_{-1}^1 y^2\,dy\right)$$

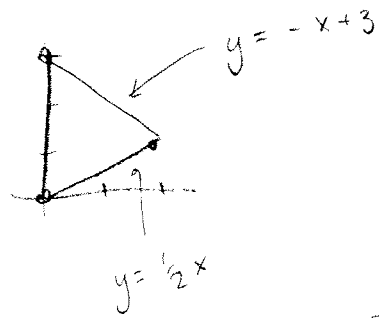
$$= \frac{3}{4}\left(\frac{x^3}{3}\Big|_0^2\right)\left(\frac{2}{3}\right) = \frac{3}{4}\cdot\frac{8}{3}\cdot\frac{2}{3} = 4/3$$

$$\bar{y} = \frac{1}{m}\iint x y^3 = \frac{3}{4}\left(\int_0^2 x\,dx\right)\left(\int_{-1}^{1} y^3\,dy\right)$$

$$= \frac{3}{4}(2)\cdot \frac{y^4}{4}\Big|_{-1}^{1} = 0$$

So $(\bar{x}, \bar{y}) = (4/3, 0)$

16.5 #5



$$m = \int_0^2 \int_{\frac{1}{2}x}^{-x+3} x+y\,dy\,dx$$

$$= \int_0^2 xy + \frac{y^2}{2}\Big|_{\frac{1}{2}x}^{-x+3} dx = \int_0^2 x(-x+3) + \frac{(-x+3)^2}{2} - x\cdot\frac{1}{2}x - \frac{(\frac{1}{2}x)^2}{2}$$

$$= \int_0^2 -x^2 + \cancel{3x} + \cancel{\frac{x^2}{2}} - \cancel{3x} + \frac{9}{2} - \cancel{\frac{x^2}{2}} - \frac{x^2}{8}$$

$$= -\frac{x^3}{3} + \frac{9}{2}x - \frac{x^3}{24}\Big|_0^2 = -\frac{8}{3} + 9 - \frac{8}{24} = 6$$

$$\bar{x} = \frac{1}{6}\int_0^2 x \int_{\frac{1}{2}x}^{-x+3} x+y\,dy\,dx = \frac{1}{6}\int_0^2 -x^3 + \frac{9}{2}x - \frac{x^3}{8}\,dx = \frac{1}{6}\left(-\frac{x^4}{4} + \frac{9}{4}x^2 - \frac{x^4}{32}\Big|_0^2\right)$$

$$= \frac{1}{6}\left(-4 + 9 - \frac{1}{2}\right) = \frac{1}{6}\cdot\frac{9}{2} = 3/4$$

$$\bar{y} = \frac{1}{6}\int_0^2 \int_{\frac{1}{2}x}^{-x+3} yx + y^2\,dy\,dx = \frac{1}{6}\int_0^2 \frac{xy^2}{2} + \frac{y^3}{3}\Big|_{\frac{1}{2}x}^{-x+3} dx$$

$$= \frac{1}{6}\int_0^2 \frac{x}{2}(-x+3)^2 + \frac{1}{3}(-x+3)^3 - \frac{x}{2}\cdot\left(\frac{x}{2}\right)^2 - \frac{1}{3}\left(\frac{x}{2}\right)^3\Big)\,dx$$

$$= \frac{1}{6}\int_0^2 \frac{x^3}{2} - 3x^2 + \frac{9}{2}x + \frac{-1}{3}x^3 - \frac{9}{3}x^2 + \frac{27}{3}x - \frac{27}{3} - \frac{x^3}{8} - \frac{x^3}{24}\,dx$$

$$= \frac{1}{6}\left(\frac{x^4}{8} - x^3 + 9x^2 - \frac{x^4}{12} - x^3 + \frac{9}{2}x^2 - 9x - \frac{x^4}{24} - \frac{x^4}{96}\right)\Big|_0^2$$

$$= \frac{1}{6}\left(2 - 8 + 36 - \frac{16}{12} - 8 + 18 - \cancel{18} - \frac{16}{24} - \frac{16}{96}\right) = \frac{1}{6}(\text{whatever})$$

16.6 #2

$$\iiint_E xz - y^3 \, dV \qquad E = [-1,1] \times [0,2] \times [0,1]$$

lets try $\displaystyle\int_{-1}^{1}\int_0^2\int_0^1 xz - y^3 \, dz\,dy\,dx$

$$= \int_{-1}^{1}\int_0^2 \frac{xz^2}{2} - zy^3 \Big|_0^1 \, dy\,dx$$

$$= \int_{-1}^{1}\int_0^2 \frac{x}{2} - y^3 \, dy\,dx = \int_{-1}^{1} \frac{xy}{2} - \frac{y^4}{4} \Big|_0^2 = \int_{-1}^{1} x - 4 \, dx$$

$$= \cancel{\int} \frac{x^2}{2} - 4x \Big|_{-1}^{1} = \frac{1}{2} - 4 - \left(\frac{1}{2} + 4\right) = -8$$

16.6 #5

$$\int_0^3\int_0^1\int_0^{\sqrt{1-z^2}} z e^y \, dx\,dz\,dy = \int_0^3\int_0^1 z e^y \sqrt{1-z^2} \, dz\,dy$$

$$= \left(\int_0^3 e^y \, dy\right)\left(\int_0^1 z\sqrt{1-z^2} \, dz\right) \qquad \begin{array}{l} u = 1-z^2 \\ du = -2z\,dz \end{array}$$

$$= (e^3 - 1)\left(-\frac{1}{2}\int_{z=0}^{z=1} u^{1/2} \, du\right)$$

$$= (e^3-1)\left(-\frac{1}{2}\,\frac{2}{3}u^{3/2}\Big|_{z=0}^{1}\right) = (e^3-1)\left(-\frac{1}{3}(1-z^2)^{3/2}\Big|_0^1\right)$$

$$= (e^3-1)\left(+\frac{1}{3}\right) = \frac{e^3-1}{3}.$$