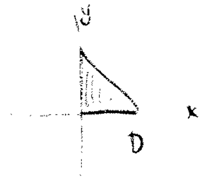
272 HW #4

16.6 # 15, 17

16.7 # 2, 10, 18

16.6 #15

$$\iiint_T x^2\, dV$$



plane has normal $\langle 1,1,1 \rangle$, thru $(1,0,0)$, so it's

$(x-1)+y+z=0$

$z=-y-x+1=1-x-y$

$$= \iint_D \int_0^{1-x-y} x^2\, dz\, dA$$

$$\iint_D dA = \int_0^1 \int_0^{1-x} dy\, dx$$

$$= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} x^2\, dz\, dy\, dx$$

$$= \int_0^1 x^2 \int_0^{1-x} 1-x-y\, dy\, dx = \int_0^1 x^2 \left(y - xy - \frac{y^2}{2} \right)\Big|_0^{1-x} dx$$

$$= \int_0^1 x^2 \left((1-x) - x(1-x) - \tfrac{1}{2}(1-x)^2 \right) dx$$

$$= \int_0^1 x^2 \left(1 - x - \cancel{x} + x^2 - \frac{1}{2} + \cancel{x} - \frac{x^2}{2} \right) dx$$

$$= \int_0^1 x^2 \left(\frac{1}{2} - x + \frac{x^2}{2} \right) dx = \int_0^1 \frac{x^2}{2} - x^3 + \frac{x^4}{2}\, dx$$

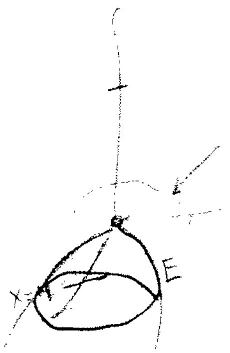
$$= \frac{x^3}{6} - \frac{x^4}{4} + \frac{x^5}{10}\Big|_0^1 = \frac{1}{6} - \frac{1}{4} + \frac{1}{10}$$

$$= \frac{10}{60} - \frac{15}{60} + \frac{6}{60} = \frac{1}{60}$$

16.6 #17

$$\iiint_E x\,dV = \iint_D \int_0^4 x\,dx\,dA \qquad \iint_D dA = \int_0^{2\pi}\int_0^1 r\,dr\,d\theta$$

$(r^2 = y^2+z^2)$



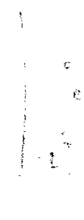
$$= \int_0^{2\pi}\int_0^1 \int_{4r^2}^4 x\,r\,dx\,dr\,d\theta$$

$$= \left(\int_0^{2\pi} d\theta\right) \int_0^1 r \int_{4r^2}^4 x\,dx\,dr = 2\pi \int_0^1 r \left.\frac{x^2}{2}\right|_{4r^2}^4 dr$$

$$= 2\pi \int_0^1 8r - 8r^5\,dr = 16\pi\left(\frac{r^2}{2} - \frac{r^6}{6}\right)\Big|_0^1$$

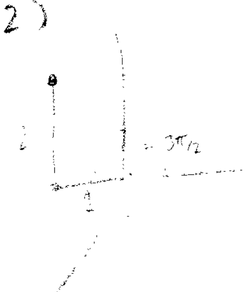
$$= 16\pi\left(\frac{1}{2} - \frac{1}{6}\right) = \frac{16}{3}\pi$$

16.7 #2 a $(1, \pi, e)$



it's $(x,y,z) = (-1, 0, e)$

b. $(1, 3\pi/2, 2)$



it's $(x,y,z) = (0,-1,2)$

16.7 #10a

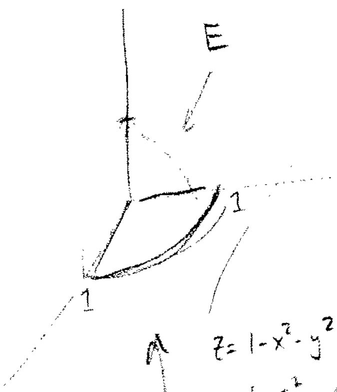
$3x+2y+z=6$

$3r\cos\theta + 2r\sin\theta + z = 6$

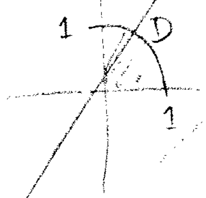
b $-x^2 - y^2 + z^2 = 1$

$-r^2 + z^2 = 1$

16.7 #18



$z = 1 - x^2 - y^2$
$= 1 - r^2$



$$\iiint_E \sqrt{x^2+y^2}\, dV = \iiint_E r\, dV$$

$$= \iint_D \int_0^{1-r^2} r\, dz\, dA$$

wrong problem!

$$= \int_0^{\pi/2}\int_0^1\int_0^{1-r^2} r\, dz\, r\, dr\, d\theta$$

$$= \int_0^{\pi/2} d\theta \cdot \left(\int_0^1 r^2 \int_0^{1-r^2} dz\, dr \right)$$

$$= \pi/2 \cdot \int_0^1 r^2 (1-r^2)\, dr$$

$$= \pi/2 \int_0^1 r^2 - r^4\, dr = \pi/2 \left(\frac{r^3}{3} - \frac{r^5}{5} \right)\Big|_0^1$$

$$= \pi/2 \left(1/3 - 1/5 \right) = \pi/2 \left(\frac{2}{15} \right) = \pi/15$$

16.7 #18

E = this solid,
D as above

$$\iiint_E x^3 + xy^2\, dV$$

$$= \iint_D \int_0^{1-r^2} x(r^2)\, dz\, dA = \int_0^{\pi/2}\int_0^1\int_0^{1-r^2} r\cos\theta\, r^2\, r\, dz\, dr\, d\theta$$

$$= \left(\int_0^{\pi/2} \cos\theta\, d\theta\right) \left(\int_0^1 r^4 \int_0^{1-r^2} dz\, dr\right) = \left(\sin\theta \Big|_0^{\pi/2}\right) \int_0^1 r^4 (1-r^2)\, dr$$

$$= 1 \cdot \int_0^1 r^4 - r^6\, dr = r^5/5 - r^7/7 \Big|_0^1 = \frac{1}{5} - \frac{1}{7} = \frac{2}{35}$$