Math 272 HW #6

16.8 22, 23

16.9 2, 4, 7

H (just the top)

16.8 #22

$$\iiint_H 9 - x^2 - y^2 \, dV$$

H is
$$0 \le \rho \le 3$$
$$0 \le \theta \le 2\pi$$
$$0 \le \phi \le \pi/2$$

$$\int_0^{\pi/2} \int_0^{2\pi} \int_0^3 (9 - x^2 - y^2)\, \rho^2 \sin\phi \; d\rho \, d\theta \, d\phi$$

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\phi$

$$= \iiint (9 - \rho^2 \sin^2\phi \cos^2\theta - \rho^2 \sin^2\phi \sin^2\theta)\; \rho^2 \sin\phi \; d\rho \, d\theta \, d\phi$$

$$= \iiint (9 - \rho^2 \sin^2\phi)\, \rho^2 \sin\phi \quad d\rho \, d\theta \, d\phi$$

$$= 2\pi \int_0^{\pi/2} \int_0^3 9\rho^2 \sin\phi - \rho^4 \sin^3\phi \; d\rho \; d\phi$$

$\int_0^{2\pi} d\theta = 2\pi$

$$= 2\pi \left(9 \int_0^{\pi/2} \sin\phi \int_0^3 \rho^2 - \int_0^{\pi/2} \sin^3\phi \int_0^3 \rho^4 \right)$$

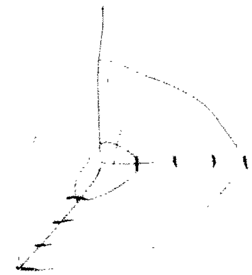
$$= 2\pi \left(9 \cdot -\cos\phi \Big|_0^{\pi/2} \cdot \frac{\rho^3}{3}\Big|_0^3 - \left(-\frac{1}{3}(2+\sin^2\phi)\cos\phi\right)\Big|_0^{\pi/2} \cdot \frac{\rho^5}{5}\Big|_0^3 \right)$$

$$= 2\pi \left(9 \cdot 1 \cdot 9 - (0 + \tfrac{1}{3}(2)\cdot 1) \cdot 3^5/5 \right)$$

$$= 2\pi \cdot \left(81 - \tfrac{2}{5} \cdot 81\right) = 2\pi \cdot \tfrac{3}{5} \cdot 81$$

16.8 #23

$$\iiint_E z \, dV$$



$1 \le \rho \le 2$

$0 \le \theta \le \pi/2$

$0 \le \phi \le \pi/2$

$$= \int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 z \ \rho^2 \sin\phi \ d\rho \, d\theta \, d\phi$$

$$= \iiint \rho \cos\phi \ \rho^2 \sin\phi \ d\rho \, d\theta \, d\phi$$

$$= \int_0^{\pi/2} d\theta \int_0^{\pi/2} \cos\phi \sin\phi \, d\phi \int_1^2 \rho^3 \, d\rho$$

$u = \sin\phi$

$du = \cos\phi \, d\phi$

$$= \pi/2 \cdot \left. \frac{\sin^2\phi}{2} \right|_0^{\pi/2} \cdot \left. \frac{\rho^4}{4} \right|_1^2$$

$$= \pi/2 \cdot \frac{1}{2} \cdot \left(\frac{2^3 - 1}{4} \right) = \frac{15\pi}{16}$$

16.9 #2

$x = uv$

$y = u/v$

Jacobian: $\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} v & 1/v \\ u & -u/v^2 \end{vmatrix} = -\frac{u}{v} - \frac{u}{v}$

$$= -2\frac{u}{v}$$

16.9 #4

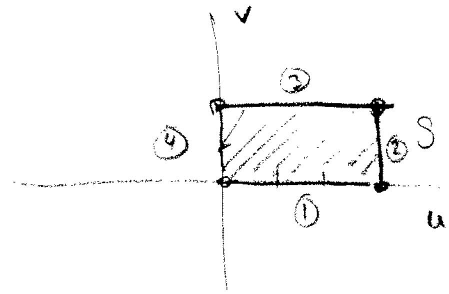
$x = e^{s+t}$

$y = e^{s-t}$

Jacobian: $\begin{vmatrix} \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} \end{vmatrix} = \begin{vmatrix} e^{s+t} & e^{s-t} \\ e^{s+t} & -e^{s-t} \end{vmatrix} = -e^{s+t}e^{s-t} - e^{s-t}e^{s+t}$

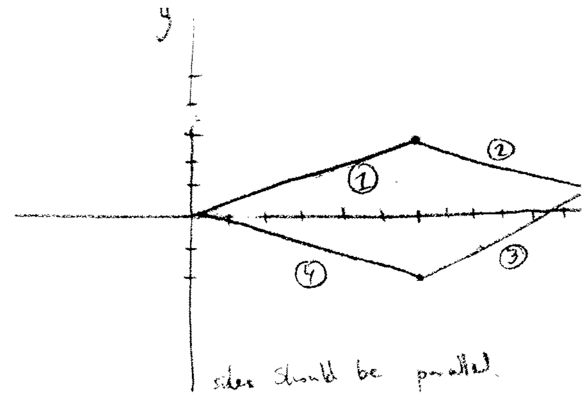
$$= -e^{2s} - e^{2s} = -2e^{2s}$$

16.9 #7



$x = 2u + 3v$

$y = u - v$



(1): $v = 0$, u goes 0 to 3

so $x = 2u$, $y = u$, so $x = 2y$. $y = \frac{1}{2}x$.

x goes 0 to 6.

(2) $u = 3$ v goes 0 to 2

so $x = 6 + 3v$, $y = 3 - v$

$v = 3 - y$, so $x = 6 + 3(3 - y) = 6 + 9 - 3y = 15 - 3y$

$y = -\frac{1}{3}x + 5$

x goes 6 to 12

(3) $v = 2$, u goes 0 to 3

so $x = 2u + 6$, $y = u - 2$ $u = y + 2$

$x = 2(y+2) + 6 = 2y + 10$ so $y = +\frac{1}{2}x - 5$

y goes -2 to 1

(4) connects it up