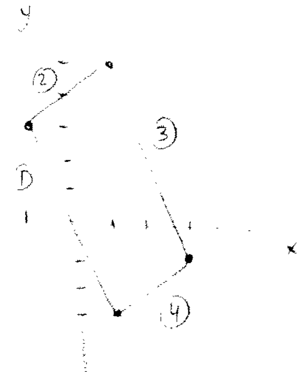
272 HW #7

16.9 12, 17a

17.1 4, 9, 25



16.9 #12

$$\iint_R 4x + 8y \, dA$$

$x = \frac{1}{4}u + \frac{1}{4}v = \frac{1}{4}(u+v)$

$y = -\frac{3}{4}u + \frac{1}{4}v = \frac{1}{4}(-3u+v)$

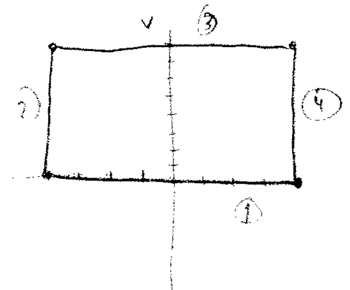
1. $y = -3x \longrightarrow -\frac{3}{4}u + \frac{1}{4}v = -3\left(\frac{1}{4}u + \frac{1}{4}v\right)$

$-3u + v = -3u - 3v$

$v = 0$, so $x = \frac{1}{4}u$, $u = 4x$

x goes -1 to 1

so u goes -4 to 4.



2. $y = x + 4 \longrightarrow \frac{1}{4}(-3u+v) = \frac{1}{4}(u+v) + 4$

$-3u + v = u + v + 16$

$u = -4$

so $x = -1 + \frac{1}{4}v$

$v = 4x + 4$

x goes -1 to 1,

so v goes 0 to 8

3: $y = -3x + 8$

$\frac{1}{4}(-3u+v) = \frac{-3}{4}(u+v) + 8$

$-3u + v = -3u - 3v + 32$

$4v = 32$

$v = 8$

so $x = \frac{1}{4}u + 2$

$u = 4x - 8$

x goes 1 to 3,

so u goes -4 to 4

4. $y = x - 4 \longrightarrow u = 4$ (as in 2)

so $v = 4x - 4$

x goes 1 to 3,

so v goes 0 to 8

Jacobian:

$$\begin{vmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{vmatrix} = \frac{1}{16} + \frac{3}{16}$$

$$= \frac{4}{16} = \frac{1}{4}$$

So $\displaystyle\iint_R 4x+8y\,dA = \int_{-4}^{4}\int_0^8 \left(u+v+2(-3u+v)\right)\tfrac{1}{4}\,dv\,du$

$$= \frac{1}{4}\int_{-4}^{4}\int_0^8 -5u+3v\,dv\,du = \frac{1}{4}\int_{-4}^{4} -5uv+\frac{3}{2}v^2\Big|_0^8\,du$$

$$= \frac{1}{4}\int_{-4}^{4} -40u+\frac{3}{2}\cdot 64\,du = \frac{1}{4}\left(-\frac{40u^2}{2}+3\cdot 32\,u\Big|_{-4}^{4}\right)$$

$$= \frac{1}{4}\left(3\cdot 32\cdot 8\right) = 6\cdot 32 = 192$$

16.9 #17a

$\displaystyle\iiint_E dV$, E is enclosed by $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$,

$x = au \quad y = bv \quad z = cw$

~~The transformation:~~
The ellipsoid becomes $\dfrac{(au)^2}{a^2}+\dfrac{(bv)^2}{b^2}+\dfrac{(cw)^2}{c^2}=1$

$u^2+v^2+w^2=1$ the unit sphere S.

Jacobian is: $\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = abc$

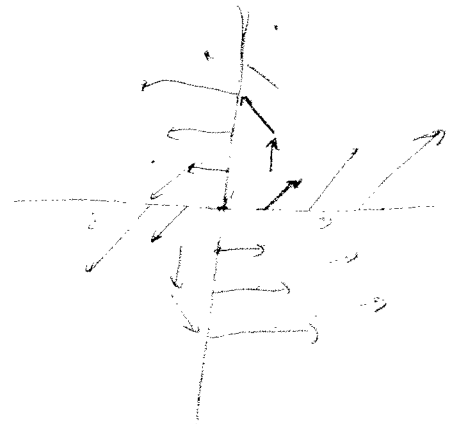
vol. of unit sphere.

so $\displaystyle\iiint_E dV = \iiint_S |abc|\,du = |abc| \iiint_S dV$

$$= |abc|\cdot\frac{4}{3}\pi$$

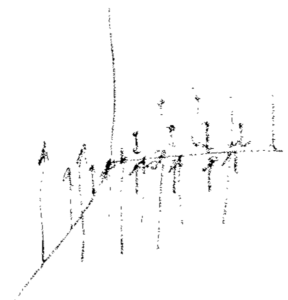
17.1 #4

$\vec{F}(x,y) = \langle x-y, x \rangle$



17.1 #9

$\vec{F}(x,y,z) = x\vec{k} = \langle 0, 0, x \rangle$



17.1 #25

$f(x,y) = x^2 - y$

$\nabla f = \langle 2x, -1 \rangle$