Math 231 HW #10

6.2 #15,

7.1 #7, 49

7.2 #10a, 34

6.2 #15 If $A \subseteq B$ then $B^c \subseteq A^c$

Pf let $x \in B^c$, we'll show $x \in A^c$.

Since $x \in B^c$, then $x \notin B$. But $A \subseteq B$, so $x \notin A$.

Thus $x \in A^c$ as desired.

7.1 #7ab

a) $F(\{1,3,4\}) = 1$

b) $F(\emptyset) = 0$

c) $F(\{2,3\}) = 0$

d) $F(\{2,3,4,5\}) = 0$

7.1 #49 $F(F^{-1}(C)) \subseteq C$

Pf let $y \in F(F^{-1}(C))$, we'll show $y \in C$.

Since $y \in F(F^{-1}(C))$, $\exists\, x \in F^{-1}(C)$ with $y = F(x)$.

Since $x \in F^{-1}(C)$ we have $F(x) \in C$,

but $y = F(x)$ so $y \in C$ as desired.

7.2 #10a $\quad f: \mathbb{Z} \to \mathbb{Z}, \quad f(n) = 2n$

it is one to one:

let $x_1, x_2 \in \mathbb{Z}$ with $f(x_1) = f(x_2)$. we'll show $x_1 = x_2$.

$f(x_1) = f(x_2)$ means $2x_1 = 2x_2$ so $x_1 = x_2$ as desired.

7.2 #34 $\quad \log_b(xy) = \log_b x + \log_b y$

Pf let $l = \log_b x$, $k = \log_b y$, so $b^l = x$, $b^k = y$ since $\log_b x$ is the inverse of b^x.

Then $b^k \cdot b^l = xy$

so $b^{l+k} = xy$

so $\log_b xy = l + k$ since $\log_b x$ is the inverse of b^x

so $\log_b xy = \log_b x + \log_b y$ as desired.