Math 231 HW #3

3.2 # 3ad, 26

4.1 # 4, 24, 33

3.~~2~~ #3ad a. $\forall$ fish x, x has gills.

Negation: $\exists$ a fish x s.t. x does not have gills

~~#~~d. $\exists$ a band b s.t. b won at least 10 Grammies

Negation: $\forall$ bands b, b won less than 10 Grammies.

3.2 ~~3.3~~ #26 $\forall$ real #s x, if $x^2 \geq 1$ then $x > 0$

or $\forall x \in \mathbb{R}$, $x^2 \geq 1 \rightarrow x > 0$

Converse: $\forall x \in \mathbb{R}$, $x > 0 \rightarrow x^2 \geq 1$

inverse: $\forall x \in \mathbb{R}$, $x^2 < 1 \rightarrow x \leq 0$

contrapos: $\forall x \in \mathbb{R}$, $x \leq 0 \rightarrow x^2 < 1$

4.1 #4 Thm $\exists\, m,n \in \mathbb{Z}$ with $m > 1$ and $n > 1$ and $\frac{1}{m} + \frac{1}{n} \in \mathbb{Z}$.

Pf Let $m = n = 2$, then $\frac{1}{m} + \frac{1}{n} = \frac{1}{2} + \frac{1}{2} = 1 \in \mathbb{Z}$. $\square$

4.1 #24 "the negative of an even integer is even"

Thm $\forall n \in \mathbb{Z}$, if n is even then $-n$ is even.

Pf Let $n \in \mathbb{Z}$ be even, so $\exists k \in \mathbb{Z}$ s.t. $n = 2k$. We'll show $-n$ is even.

we have $-n = -2k = 2(-k)$, so $-n$ is even as desired.

4.1 # 33 Thm If n is any even integer, then $(-1)^n = 1$.

Pf Let $n \in \mathbb{Z}$ be even. Then $\exists k \in \mathbb{Z}$ s.t. $n = 2k$.

Then $(-1)^n = (-1)^{2k} = ((-1)^2)^k = (1)^k = 1$

since any power of 1 is 1. So $(-1)^n = 1$ as desired.