

Math 231 HW #4

Section 4.2: 6, 17, 35

4.3: 15, 29

4.2 #6

$$\cancel{3824}x = 320.5\overline{429}$$

$$10x = 3205.\overline{429}$$

$$10000x = 3205429.\overline{429}$$

$$\hookrightarrow 10000x - 10x = 3205429.\overline{429} - 3205.\overline{429}$$

$$9990x = 3202224$$

$$\text{so } x = \frac{3202224}{9990}$$

4.2 #17 Thm $\forall x, y \in \mathbb{Q}, x - y \in \mathbb{Q}$.

Pf let $x, y \in \mathbb{Q}$, so $\exists a, b, c, d \in \mathbb{Z}$ with

$$x = \frac{a}{b} \quad \text{and} \quad y = \frac{c}{d} \quad \text{with} \quad b \neq 0 \quad \text{and} \quad d \neq 0.$$

$$\text{Then } x - y = \frac{a}{b} - \frac{c}{d} = \frac{ad - cb}{bd} \quad \text{and} \quad bd \neq 0$$

$$\text{so } x - y \in \mathbb{Q}.$$

4.2 #35

The first sentence assumes the result to be proved.

4.3 #15 Thm $\forall a, b, c \in \mathbb{Z}$, if $a|b$ and $a|c$ then $a|b+c$.

Pf Let $a, b, c \in \mathbb{Z}$, and let $a|b$ and $a|c$.
We'll show $a|b+c$.

Since $a|b$ and $a|c$ we know $\exists k, l \in \mathbb{Z}$ with
 $b = ka$ and $c = la$.

Then $b+c = ka + la = (k+l)a$ and so $a|b+c$. Shown.

4.3 #29 Thm $\forall a, b \in \mathbb{Z}$, if $a|b$ then $a^2|b^2$.

Pf Let $a, b \in \mathbb{Z}$, and let $a|b$.

Then $\exists k \in \mathbb{Z}$ with $b = ka$, and so $b^2 = (ka)^2 = k^2 a^2$.

But $b^2 = k^2 a^2$ means $a^2|b^2$ since $k^2 \in \mathbb{Z}$, as desired.