Math 231 HW #6

Section 4.5 # ~~[illegible]~~ 12, 15, 21

4.6 # 7, 11

4.5 #12 Thm If n is even, then $\lfloor n/2 \rfloor = n/2$.

Pf Let n be even, then $\exists k \in \mathbb{Z}$ s.t. $n = 2k$,

then
$$\lfloor n/2 \rfloor = \lfloor 2k/2 \rfloor$$
$$= \lfloor k \rfloor$$
$$= k \quad \text{since } k \in \mathbb{Z}$$
$$= n/2 \quad \text{as desired.}$$

4.5 #15 Thm $\forall x \in \mathbb{R}$, $\lfloor x-1 \rfloor = \lfloor x \rfloor - 1$.

Pf Let $n = \lfloor x-1 \rfloor$, so

$n \le x-1 < n+1$,

so $n+1 \le x < n+2$,

so $n+1 = \lfloor x \rfloor$

so $n = \lfloor x \rfloor - 1$, so $\lfloor x-1 \rfloor = \lfloor x \rfloor - 1$ as desired.

4.5 #21

Thm $\forall$ odd n, $\lceil n/2 \rceil = \frac{n+1}{2}$

Pf Let n be odd, so $n = 2k+1$ for $k \in \mathbb{Z}$.

Then $\lceil n/2 \rceil = \lceil \frac{2k+1}{2} \rceil = \lceil k + \frac{1}{2} \rceil = k+1$ since $k \in \mathbb{Z}$.

but $k = \frac{n-1}{2}$, so $k+1 = \frac{n}{2} - \frac{1}{2} + 1 = \frac{n}{2} + \frac{1}{2} = \frac{n+1}{2}$,

so $\lceil n/2 \rceil = \frac{n+1}{2}$.

4.6 #7

Thm There is no least positive ~~integer~~ rational

Pf by contradiction, assume there is a least pos. ~~integer~~ rational.

so $\exists x \in \mathbb{Q}$ s.t. $\forall y \in \mathbb{Q}$, $x \le y$.

let $z = \frac{x}{2}$, then $z \in \mathbb{Q}$, and $z < x$.

But $x \le z$ since x is the smallest, which is a contradiction.

4.6 # 11 Thm $\forall x, y$ with $x \in \mathbb{Q}, x \neq 0$, $y \notin \mathbb{Q}$, $xy \notin \mathbb{Q}$.

Pf by contradiction: Assume $x \in \mathbb{Q}$, $x \neq 0$, $y \notin \mathbb{Q}$, and $xy \in \mathbb{Q}$.

~~Then we have a, b, c, d ∈~~

Since a quotient of rationals is rational we have

$\frac{xy}{x} \in \mathbb{Q}$, but $\frac{xy}{x} = y$ so $y \in \mathbb{Q}$

which contradicts $y \notin \mathbb{Q}$.