Math 231 HW # 7

4.6 #20, 28b

4.7 #8

~~4~~ 5.1 #22

5.2 #6

4.6 #20 Thm $\forall x,y \in \mathbb{R}$, $x+y < 50 \rightarrow$ ~~$x<25$ or $y<25$~~ $x<25$ or $y<25$

Pf Will prove the contrapositive:

$$x \geq 25 \text{ and } y \geq 25 \rightarrow x+y \geq 50$$

Assume $x \geq 25$ and $y \geq 25$, then

$$x+y \geq 25+25 = 50$$

so $x+y \geq 50$ as desired.

4.~~3~~6 #28b

Thm $\forall m,n \in \mathbb{Z}$, mn is even $\rightarrow$ m is even or n is even.

Pf FSOC, ~~assume~~ let mn be even and assume m is odd and n is odd.

Then mn is (odd) × (odd) which is odd,

but mn is even, which is a contradiction.

4.7 #8 "Difference of any two irrationals is irrational"

This is false:

$$\pi - \pi = 0$$

irrationals rational

a less-stupid example:

$$\pi - (\pi - 1) = 1$$

irrationals rational

5.1 #22

$$\prod_{j=0}^{4} (-1)^j = (-1)^0 (-1)^1 (-1)^2 (-1)^3 (-1)^4$$
$$= 1 \cdot (-1) \cdot 1 \cdot (-1) \cdot 1$$
$$= 1$$

5.2 #6 Thm $\forall n \in \mathbb{Z}^+$, $\quad 2 + 4 + \dots + 2n = n^2 + n$

Pf By induction on n:

Basis step $(n = 1)$ We need $2 = 1^2 + 1$ which is clear.

Inductive step Assume $2 + 4 + \dots + 2k = k^2 + k$,

we'll show $2 + \dots + 2k + 2(k+1) = (k+1)^2 + (k+1)$.

We have

$$2 + 4 + \dots + 2k + 2(k+1) = k^2 + k + 2(k+1) \qquad \text{by IH}$$
$$= k^2 + 3k + 2$$
$$= k^2 + 2k + 1 + k + 1$$
$$= (k+1)^2 + (k+1) \qquad \text{as desired.}$$