

Math 235 HW #10

4.4 #4, 18, 19, 21, 25

#4

$$A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad \text{so} \quad V = \sqrt{|\det A^T A|}$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 4 & 1 \\ 2 & 2 & 1 & -1 \\ 4 & 1 & 10 & 3 \\ 1 & -1 & 3 & 3 \end{bmatrix}$$

$$\det A^T A = 9 \quad (\text{computer})$$

$$\text{so} \quad V = \sqrt{9} = 3.$$

#18

$$V = |\det A| \cdot (\text{original volume})$$

$$\text{here, } T(\vec{e}_1) = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \quad T(\vec{e}_2) = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

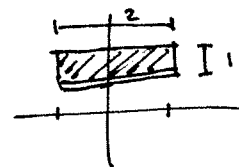
$$\text{so} \quad A = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}, \quad \det A = 12 + 4 = 16$$

$$\text{original volume is } 1, \quad \text{so} \quad V = 16 \cdot 1 = 16.$$

#19

Same  $A$ , this time original volume is  $2 \cdot 1 = 2$

$$\text{so} \quad V = 16 \cdot 2 = 32$$



#21

Same  $A$ , here it's a circle, radius 3,

$$\text{so original volume is } \pi r^2 = 9\pi$$

$$\text{so} \quad V = 16 \cdot 9\pi = 144\pi$$

#25  $T(\vec{e}_1) = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$   $T(\vec{e}_2) = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$   $T(\vec{e}_3) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

so  $A = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 0 & 1 \\ 4 & 3 & 0 \end{bmatrix}$

$$\det A = \begin{vmatrix} 1 & -2 & 0 \\ 3 & 0 & 1 \\ 4 & 3 & 0 \end{vmatrix} = - \begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix} = - (3 + 8) = -11$$

so  $|\det A| = 11$ .

Original shape is sphere, radius 4, so

orig. volume is  $\frac{4}{3} \pi r^3 = \frac{256}{3} \pi$

so  $V = 11 \cdot \frac{256}{3} \pi = \frac{2816}{3} \pi$