Math 235 HW #11

5.1 # 3, 8, 19, 24, 26

5.1 #3 $A = \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix}$

$$|A - \lambda I| = \begin{vmatrix} -1-\lambda & -2 \\ 4 & 5-\lambda \end{vmatrix} = (-1-\lambda)(5-\lambda) + 8$$

char polyn.

$$= -5 - 4\lambda + \lambda^2 + 8 = \boxed{\lambda^2 - 4\lambda + 3} = (\lambda-1)(\lambda-3)$$

$\boxed{\lambda = 1,\ 3}$ eigenvalues

$\lambda = 1$ $A - I$: $\left[\begin{array}{cc|c} -2 & -2 & 0 \\ 4 & 4 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\begin{bmatrix} -r \\ r \end{bmatrix} = r\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

r

eigenvector $= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\lambda = 3$ $A - 3I$: $\left[\begin{array}{cc|c} -4 & -2 & 0 \\ 4 & 2 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1/2 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\begin{bmatrix} -\frac{1}{2}r \\ r \end{bmatrix} = r\begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$

r

eigenvector $= \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$.

5.1 #8 $A = \begin{bmatrix} -1 & 0 & 0 \\ -4 & 2 & -1 \\ 4 & 0 & 3 \end{bmatrix}$

char poly

$$|A - \lambda I| = \begin{vmatrix} -1-\lambda & 0 & 0 \\ -4 & 2-\lambda & -1 \\ 4 & 0 & 3-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} -1-\lambda & 0 \\ 4 & 3-\lambda \end{vmatrix} = \boxed{(2-\lambda)(-1-\lambda)(3-\lambda)}$$

$\lambda = 2, -1, 3$ eigenvalues

$\underline{\lambda = 2}$

$$\left[\begin{array}{ccc|c} -3 & 0 & 0 & 0 \\ -4 & 0 & -1 & 0 \\ 4 & 0 & 1 & 0 \end{array}\right] \to \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ \uparrow \\ r \end{array} \qquad \begin{array}{l} x = 0 \\ y = r \\ z = 0 \end{array} \to r\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

eigenvector is $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\underline{\lambda = -1}$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -4 & 3 & -1 & 0 \\ 4 & 0 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ \uparrow \\ r \end{array} \to \begin{bmatrix} -r \\ -r \\ r \end{bmatrix} = r\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

eigenvector $= \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

$\underline{\lambda = 3}$

$$\left[\begin{array}{ccc|c} -4 & 0 & 0 & 0 \\ -4 & -1 & -1 & 0 \\ 4 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{bmatrix} 0 \\ -r \\ r \end{bmatrix} = r\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

eigenvector $= \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

$\underline{5.1 \ \# 19}$ $\quad T([x_1, x_2, x_3]) = [x_1 + x_3, x_2, x_1 + x_3]$

has matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)\Big((1-\lambda)(1-\lambda) - 1\Big)$$

$$= (1-\lambda)(1 - 2\lambda + \lambda^2 - 1) = (1-\lambda)(\lambda^2 - 2\lambda)$$

$$= (1-\lambda)(\lambda - 2)\lambda$$

$\lambda = 1, 2, 0$

$\underline{\lambda = 1}$

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right] \to \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ \uparrow \\ r \end{array} \qquad \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix} = r\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

eigenvector $= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\underline{\lambda = 2}$

$$\left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right] \to \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ \begin{array}{cccc} & & \uparrow & \\ & & r & \end{array} \end{array} \qquad \begin{bmatrix} r \\ 0 \\ r \end{bmatrix} = r \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{eigenvector} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$\underline{\lambda = 0}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right] \to \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ \begin{array}{cccc} & & \uparrow & \\ & & r & \end{array} \end{array} \qquad \begin{bmatrix} -r \\ 0 \\ r \end{bmatrix} = r \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{eigenvector} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

5.1 #24 Show $S = \{ \vec{v} \in V \mid T(\vec{v}) = \lambda \vec{v} \}$ is a subspace:

closed under +: Take $\vec{v}, \vec{w} \in S$, so $T(\vec{v}) = \lambda \vec{v}$, $T(\vec{w}) = \lambda \vec{w}$,
we need $\vec{v} + \vec{w} \in S$, i.e. $T(\vec{v} + \vec{w}) = \lambda(\vec{v} + \vec{w})$.

Pf $T(\vec{v} + \vec{w}) = T(\vec{v}) + T(\vec{w})$ (T is linear)

$= \lambda \vec{v} + \lambda \vec{w}$ ($\vec{v}, \vec{w} \in S$)

$= \lambda(\vec{v} + \vec{w})$ (scalar mult. is distributive)

shown.

closed under scalar mult: Take $\vec{v} \in S$, $r \in \mathbb{R}$,
we need $r\vec{v} \in S$, i.e. $T(r\vec{v}) = \lambda(r\vec{v})$

Pf $T(r\vec{v}) = r\,T(\vec{v})$ (T is linear)

$= r \lambda \vec{v}$ ($\vec{v} \in S$)

$= \lambda(r\vec{v})$ (scalars commute

shown.

5.1 # 26 $\quad T$ is the 4^{th} deriv.

e^{ax} is an eigenvector:

$$T(e^{ax}) = a^4 e^{ax} \quad \text{(chain rule 4 times)}$$

so e^{ax} is an eigenvector with $\lambda = 4$.

Similarly:

e^{-ax}: $\quad T(e^{-ax}) = (-a)^4 e^{-ax} = a^4 e^{-ax}$

so e^{-ax} has $\lambda = a^4$

$\cos ax$: $\quad T(\cos ax) = a^4 \cos ax$

so $\cos ax$ has $\lambda = a^4$

$\sin ax$: $\quad T(\sin ax) = a^4 \sin ax$

so $\sin ax$ has $\lambda = a^4$.

all have eigenvalue a^4.