

Math 235

HW #3

1.4 # 10, 44, 47

1.5 15, 16

1.4 #10

$$\left[\begin{array}{cccc|c} \textcircled{1} & 1 & 0 & 3 & 0 & -4 \\ 0 & 0 & \textcircled{1} & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \textcircled{1} & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \rightarrow x_1 + s + 3r = -4 \rightarrow x_1 = -4 - s - 3r \\ \rightarrow x_3 - r = 0 \rightarrow x_3 = r \\ \rightarrow x_5 = -2 \end{array}$$

\uparrow $x_2 = s$ \uparrow $x_4 = r$

general soln:
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -4 - s - 3r \\ s \\ r \\ r \\ -2 \end{bmatrix}$$

particular soln with $x_2 = 2, x_3 = 1$: $s = 2, r = 1$

so it's
$$\begin{bmatrix} -4 - 2 - 3 \\ 2 \\ 1 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -9 \\ 2 \\ 1 \\ 1 \\ -2 \end{bmatrix}$$

1.4 #44

$$C \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -2 \\ 0 & -6 \end{bmatrix}$$

~~adds~~

C subtracts $3 \times R1$ from $R2$, then subtracts $4 \times R1$ from $R3$.

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

So C is the product

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

1.4 #47

The row exchange is $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

the other is $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Do them both with

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 4 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 4 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.5 #15

$$2x + y + 4z = r$$

$$3x + 2y + 5z = s$$

$$-y + z = t$$

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & -1 & 1 \end{bmatrix}$$

from ~~the~~ ⁷, $A^{-1} = \begin{bmatrix} -7 & 5 & 3 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \end{bmatrix}$

$$\text{So } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 & 5 & 3 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix} = \begin{bmatrix} -7r + 5s + 3t \\ 3r - 2s - 2t \\ 3r - 2s - t \end{bmatrix}$$

1.5 #16

$$A^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}, \quad AC = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix}$$

$$\text{So } C = A^{-1}AC = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 4 & 4 \\ 12 & 11 \end{bmatrix}$$