Math 235

HW #4

1.6. #2, 8, 16, 18, 23

#2 $W = \{[x, x+1] \mid x \in \mathbb{R}\}$

let's check closed under $+$:

Take $[a, a+1], [b, b+1] \in W$, then

$$[a, a+1] + [b, b+1] = [a+b, a+b+2]$$

not in W (this needs to be 1)

so W is not closed under $+$, so not a subspace

#8 $W = \{[2x, x+y, y] \mid x, y \in \mathbb{R}\}$ is a subspace

closed under $+$: Take $[2a, a+b, b], [2a', a'+b', b'] \in W$.

Then $[2a, a+b, b] + [2a', a'+b', b'] = [2(a+a'), a+a'+b+b', b+b'] \in W$

so it is closed under $+$.

closed under $\cdot$: Take $[2a, a+b, b] \in W$, $r \in \mathbb{R}$. Then

$$r[2a, a+b, b] = [2ra, ra+rb, rb] \in W$$

so it is closed under sc. mult.

So it is a subspace

#16 A basis for the solns to $\begin{aligned} x - y &= 0 \\ 2x - 2y &= 0 \end{aligned}$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 2 & -2 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad x = r$$

$y = r$

solns are $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix} = r\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ a basis is $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

#18

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 2 & -1 & 3 & 0 \end{array}\right] \to \cdots \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & 3/5 & 0 \\ 0 & 1 & 0 & 4/5 & 0 \\ 0 & 0 & 1 & -4/5 & 0 \end{array}\right]$$

r

$$\vec{x} = \begin{bmatrix} -3/5\, r \\ -4/5\, r \\ 4/5\, r \\ r \end{bmatrix} = r\begin{bmatrix} 3/5 \\ -4/5 \\ 4/5 \\ 1 \end{bmatrix} \qquad \text{basis is } \left\{ \begin{bmatrix} 3/5 \\ -4/5 \\ 4/5 \\ 1 \end{bmatrix} \right\}$$

#23 Is $\left\{ \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \right\}$ a basis for the span?

need only check if $c_1\vec{v}_1 + c_2\vec{v}_2 = 0$ has a unique soln.

$$\left[\begin{array}{cc|c} -1 & 2 & 0 \\ 3 & 1 & 0 \\ 4 & 4 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 7 & 0 \\ 0 & 12 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

<u>does</u> have a unique soln, so it <u>is</u> a basis for the span.