

Math 235
HW #5

2.1 12, 21

2.2 ~~5 abc~~

2.3 3, 5

2.1 #12

Basis for colspace:

$$\begin{bmatrix} 2 & 3 & 1 \\ 5 & 2 & 1 \\ 1 & 7 & 2 \\ 6 & -2 & 0 \end{bmatrix} \rightarrow \dots \rightarrow \begin{bmatrix} \textcircled{1} & 0 & 1/11 \\ 0 & \textcircled{1} & 3/11 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{basis is } \left\{ \begin{bmatrix} 2 \\ 5 \\ 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 7 \\ -2 \end{bmatrix} \right\}$$

~~2.1 #21~~ 2.1 #21 $\{[-1, 2, 1], [2, -4, 3]\}$ is indep

$$\begin{bmatrix} -1 & 2 \\ 2 & -4 \\ 1 & 3 \end{bmatrix} \rightarrow \dots \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

a pivot in each col,
so it's indep.

~~2.2 #5abc~~

2.2 #5abc

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & 1 & 0 & 2 \\ 0 & 2 & 1 & 1 \end{bmatrix} \rightarrow \dots \rightarrow \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 1/3 \\ 0 & 0 & 1 & 1/3 \end{bmatrix}$$

$$\text{basis for colspace: } \left\{ \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\text{basis for row space: } \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 5/6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1/3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1/3 \end{bmatrix} \right\}$$

2.3 #3

$T(x_1, x_2, x_3) = [1, 1, 1, 1]$ is not linear:

$$T([x_1, x_2, x_3] + [y_1, y_2, y_3]) = [1, 1, 1, 1]$$

$$T([x_1, x_2, x_3]) + T([y_1, y_2, y_3]) = [1, 1, 1, 1] + [1, 1, 1, 1] = [2, 2, 2, 2]$$

not the same.

2.3 #5

$$T([4, -6]) = T(4[1, 0] + -6[0, 1])$$

$$= T(4[1, 0]) + T(-6[0, 1])$$

$$= 4T([1, 0]) - 6T([0, 1])$$

$$= 4[3, -1] - 6[-2, 5]$$

$$= [12, -4] + [12, -30] = [24, -34].$$