Math 235 HW #6

2.3 #14

2.4 #4

3.1 #4, 10, 11

<u>2.3 #14</u>

$$T([x_1, x_2]) = [2x_1 - x_2,\ x_1 + x_2,\ x_1 + 3x_2]$$

$$T(\vec{e}_1) = T([1,0]) = [2, 1, 1]$$
$$T(\vec{e}_2) = T([0,1]) = [-1, 1, 3]$$

so the matrix is $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$

<u>2.4 #4</u> Triple angle formulas:

$$\begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^3\theta - \cos\theta\sin^2\theta - 2\sin^2\theta\cos\theta & \\ -2\sin\theta\cos^2\theta - \sin\theta\cos^2\theta + \sin^3\theta & \end{bmatrix}$$

so $\cos 3\theta = \cos^3\theta - 3\sin^2\theta\cos\theta$

$\sin 3\theta = -\sin^3\theta + 3\sin\theta\cos^2\theta$

<u>3.1 #4</u> $A \star B = O$

There is no zero vector here. If there were a $\vec{O}$, then $A \star \vec{O} = A$ for all A.

But $A \star \vec{O} = O$ always, so this is impossible.

3.1 #10 The set of 2×2 matrices like $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ is

not closed under addition:

$$\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} x & 1 \\ 1 & y \end{bmatrix} = \begin{bmatrix} a+x & 2 \\ 2 & b+y \end{bmatrix},$$

which is not of the same form

(because of the 2s)

3.1 #11 n×n ~~2×2~~ diagonal matrices, like this $\begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix}$

It is a vectorspace:

closed under + : $\begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix} + \begin{bmatrix} b_1 & & 0 \\ & \ddots & \\ 0 & & b_n \end{bmatrix} = \begin{bmatrix} a_1+b_1 & & 0 \\ & \ddots & \\ 0 & & a_n+b_n \end{bmatrix}$

is diagonal

closed under sc. mult: $r\begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix} = \begin{bmatrix} ra_1 & & 0 \\ & \ddots & \\ 0 & & ra_n \end{bmatrix}$ ← is diagonal

A1, A2, S1–S4 all hold because we're using usual matrix operations.

For A3, $\vec{0}$ is the zero matrix which is diagonal

For A4, $-\begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix} = \begin{bmatrix} -a_1 & & 0 \\ & \ddots & \\ 0 & & -a_n \end{bmatrix}$ is diagonal.