Math 235

HW # 7

3.1 #19

3.2 #2, 12, 20, 22

3.1 #19 Prove $-\vec{v}$ is unique.

Pf Assume $\vec{v}+(-\vec{v})=\vec{0}$, also $\vec{v}+(-\vec{v}')=\vec{0}$.

we'll show $-\vec{v}=-\vec{v}'$.

Subtract these eqns

$$\vec{v}+(-\vec{v})-(\vec{v}+(-\vec{v}'))=\vec{0}-\vec{0}$$

$$\vec{v}-\vec{v}+(-\vec{v})-(-\vec{v}')=\vec{0}$$

$$\vec{0}+(-\vec{v})-(-\vec{v}')=\vec{0}$$

$$(-\vec{v})-(-\vec{v}')=\vec{0}$$

$$-\vec{v}=-\vec{v}' \quad \text{as desired.}$$

3.2 #2 Polys of deg 4 with 0.

It's not a subspace, not closed under +.

e.g. x^4+1 is deg 4,

$-x^4$ is deg 4,

but $(x^4+1)+(-x^4)=1$

not deg 4, and not 0.

3.2 #12 $\{1, 4x+3, 3x-4, x^2+2, x-x^2\}$
is it independent?

Easy answer: it's dependent, since these are ~~4~~ 5 things in P_2 which is only dimension 3.

Long answer: $c_1 1 + c_2(4x+3) + c_3(3x-4) + c_4(x^2+2) + c_5(x-x^2) = 0$

$$(-c_5+c_4)x^2 + (4c_2+3c_3+c_5)x + (c_1+3c_2-4c_3+2c_4)\cdot 1 = 0$$

so
$$c_4 - c_5 = 0$$
$$4c_2+3c_3+c_5 = 0$$
$$c_1+3c_2-4c_3+2c_4 = 0$$
$$\rightarrow \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & -4 & 2 & 0 & 0 \end{array}\right]$$

$\rightarrow$ will have free vars,
so $c_1 = \dots = c_5 = 0$ is not the only soln, so they're not independent.

3.2 #20 $\{x, x^2+1, (x-1)^2\}$ is it a basis for P^2?

~~There~~ They are dependent:
$$-2(x) + 1(x^2+1) + (-1)(x-1)^2 = 0$$
so it's not a basis.

3.2 #22 Find a basis for $sp\{x^2-1, x^2+1, 4, 2x-3\}$.

Which are independent? $c_1(x^2-1) + c_2(x^2+1) + c_3\cdot 4 + c_4(2x-3) = 0$

gives $\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ -1 & 1 & 4 & -3 & 0 \end{array}\right] \longrightarrow \dots \rightarrow \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ cols 1, 2, 4 are indep

A basis is $\{x^2-1, x^2+1, 2x-3\}$