Math 235 HW #8

3.3 #4, 10, 21

3.4 #2, 36

3.3 #4

$$\left[\begin{array}{ccc|c} 0 & 2 & 0 & 4 \\ 1 & 0 & 3 & -2 \\ 1 & 0 & 0 & 1 \end{array}\right] \to \cdots \to \left[\begin{array}{ccc|c} 1 & & & 1 \\ & 1 & & 2 \\ & & 1 & -1 \end{array}\right]$$

coords are $[1, 2, -1]$

3.3 #10

$\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ wrt $\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\right)$

$$\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} = c_1 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} + c_4 \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_3 & c_1 - c_2 - c_3 + c_4 \\ c_1 & 3c_3 + c_4 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & -1 & 1 & -2 \\ 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 \end{array}\right] \to \cdots \to \left[\begin{array}{cccc|c} 1 & & & & 3 \\ & 1 & & & 5 \\ & & 1 & & 1 \\ & & & 1 & 1 \end{array}\right]$$

coords are $[3, 5, 1, 1]$

3.3 #21

For $B = (x + x^2, x - x^2, 1 + x)$

coords $[3, 1, 2]$ means

$$3(x + x^2) + 1(x - x^2) + 2(1 + x)$$

$$= 3x + 3x^2 + x - x^2 + 2 + 2x$$

$$= 2x^2 + 6x + 2$$

3.4 #2 $T(f) = (f(5))^2$ is not linear.

addition would be $T(f+g) = T(f) + T(g)$

$T(f+g) = ((f+g)(5))^2 = (f(5) + g(5))^2$

$T(f)+T(g) = (f(5))^2 + (g(5))^2$ ← not the same,

so not linear.

3.4 #36 T, T' linear, show $T' \circ T$ is linear.

Addition need $T' \circ T(\vec{u}+\vec{v}) = T' \circ T(\vec{u}) + T' \circ T(\vec{v})$

Pf $T' \circ T(\vec{u}+\vec{v}) = T'(T(\vec{u}+\vec{v})) = T'(T(\vec{u}) + T(\vec{v}))$

$= T'(T(\vec{u})) + T'(T(\vec{v}))$

$= T' \circ T(\vec{u}) + T' \circ T(\vec{v})$ shown

sc. mult need $T' \circ T(r\vec{v}) = r\,T' \circ T(\vec{v})$

Pf $T' \circ T(r\vec{v}) = T'(T(r\vec{v})) = T'(rT(\vec{v}))$

$= rT'(T(\vec{v})) = r\,T' \circ T(\vec{v})$ shown