Math 235 HW #9

3.4 #1, 24a

~~3~~4.1 # 19 ~~?~~e

4.2 26, 32

3.4 #1 $T: f \to \mathbb{R}$, $T(f) = f(-4)$

last week we showed it's linear

$$\left(T(f+g) = (f+g)(-4) = f(-4) + g(-4) = T(f) + T(g) \right)$$
$$T(rf) = (rf)(-4) = r\,f(-4) = r\,T(f)$$

ker T is all f with $T(f) = 0$, i.e.

all functions with $f(-4) = 0$.

These are all fcns with x-intercept at -4.

ker (T) is not just 0, so T is not injective,

so not invertible.

3.4 #24a $T: P_3 \to P_2$, $T(p(x)) = p'(2x+1)$

$T(x^3) = 3(2x+1)^2 = 3(4x^2+4x+1) = 12x^2 + 12x + 3$

$T(x^2) = 2(2x+1) = 4x+2$

$T(x) = 1$

$T(1) = 0$

the matrix is $\begin{bmatrix} 12 & 0 & 0 & 0 \\ 12 & 4 & 0 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}$

4.1 #19e Any 3 pts in $\mathbb{R}^3$ lie in a plane, so this couldn't by itself imply that det $=0$.

So False

4.2 #26 need $\det\begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} = 0$

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda) - 6$$
$$= \lambda^2 - 3\lambda + 2 - 6$$
$$= \lambda^2 - 3\lambda - 4$$

$$\lambda^2 - 3\lambda - 4 = 0$$
$$(\lambda-4)(\lambda+1) = 0$$
$$\lambda = 4, -1.$$

4.2 #32 A, C are $n\times n$, C is invertible.

Then $\det(A) = \det(C^{-1}AC)$.

Pf Since C is square,

$$\det(C^{-1}AC) = \det(C^{-1}A)\det C$$
$$= \det(C^{-1})\det A \det C$$
$$= \frac{1}{\cancel{\det C}}(\det A)(\cancel{\det C}) = \det A$$ shown!