# Math 235 final exam topics

These are some sample problems– for more examples, see your homework problems from the appropriate section. These sample problems are not meant to be exhaustive of what you should know from each section.

Big general thing to know: The "invertible matrix theorem". For a $n \times n$ matrix $A$, the following are equivalent:

- $A$ is invertible
- $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b}$
- The rank of $A$ is $n$
- The columnspace of $A$ is $\mathbf{R}^n$
- The nullspace of $A$ is $\{\mathbf{0}\}$
- $\det A \neq 0$

## Old topics (50% of the exam)

### 1.1: Vectors

- Write $[1, 3]$ as a linear combination of $[1, 0]$ and $[2, -1]$

### 1.2: Magnitude and the dot product

- Find $||[1, 3, 2]||$
- Show that $[2, 4, -1] \perp [1, 1, 6]$

You should know the properties of the dot product from Theorem 1.3

### 1.3: Matrix multiplication, transpose

- Multiply some matrices

You should know the properties of matrix multiplication and the transpose on page 45.

### 1.4: Gauss–Jordan, elementary matrices

- Solve the system:

$$\begin{aligned} 2x - y &= 2 \\ -x + 4y &= 1 \end{aligned}$$

  using Gauss–Jordan elimination.

- Know how to express infinitely many solutions in terms of free variables.
- Give a $3 \times 3$ matrix which, when multiplied on the left, has the effect of exchanging row 1 and row 3, followed by adding 3 times row 2 to row 1.

## 1.5: Matrix inverse

- Find the inverse of:

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 5 & -3 \\ -3 & 2 & -4 \end{bmatrix}$$

- Solve the system above by inverting a matrix
- Show that $(AB)^{-1} = B^{-1}A^{-1}$

## 1.6: Subspaces and Bases

- Show that $\{[x, y, z] \mid y = 2x\}$ is a subspace
- Find a basis for $\mathrm{sp}\{[1, 0, 2], [3, 4, 1], [4, 4, 3]\}$

## 2.1: Independence and dimension

- Show that $\{[1, 2, 6, 0], [2, 3, 0, -1], [-1, 0, 0, 0]\}$ is linearly independent
- Find the dimension of $\mathrm{sp}\{[1, 0, 2], [3, 4, 1], [4, 4, 3]\}$

## 2.2: Rank of a matrix

- Find the rank of a matrix

## 2.3: Linear transformations of $\mathbf{R}^n$

- Let $T([x, y, z]) = [3x^2, 2y + z]$. Is $T$ linear?
- Find the standard matrix given by a linear transformation
- Find a $2 \times 2$ matrix which has the effect of rotating by $-90°$, then flipping over the line $y = x$.

## 3.1: Vectorspaces

The 8 vectorspace axioms will be given to you.

- Show that the set of all $n \times n$ diagonal matrices is a vectorspace

## 3.2: Properties of vectorspaces

- Show that $\{x^2, 3x - 2, x^3\}$ is linearly independent in $P_3$.
- Find the dimension of the set of all $n \times n$ diagonal matrices.

## 3.3: Coordinates

- Show that $\{x^2, 3x - 2, x^3\}$ is linearly independent in $P_3$

### 3.4: Linear transformations of vectorspaces

- Show that the second derivative is a linear transformation
- Find the kernel of the second derivative
- Is the second derivative one-to-one?
- Find the matrix for the second derivative when viewed as a transformation $T : P_5 \to P_3$

## New topics (50% of the exam)

### 4.2: Determinant

- Find the determinant of a fairly small matrix
- Use some tricks to find the determinant of bigger matrices
- Know Theorems 4.3, 4.4

### 4.4: Volume change factors

- Find the volume of the 2-box in $\mathbf{R}^3$ made by $[1, 3, 2]$ and $[1, 2, -1]$
- Find the area of the image of the unit circle under the transformation $T([x, y]) = [2x + y, -x + y]$.

### 5.1: Eigenvalues and eigenvectors

- Find eigenvalues and eigenvectors for a matrix

### 5.2: Diagonalization

- Find a diagonalization for a matrix
- Find a high power of a matrix by hand

### 6.5: Least squares

- Fit some data points with the least squares line