

Section 8.4 Math 122 HW # 9

#2, 7, 8, 14, 20

8.4 #2

$$\int_7^{\infty} \frac{1}{x^2} dx = \lim_{b \rightarrow \infty} \int_7^b x^{-2} dx = \lim_{b \rightarrow \infty} -x^{-1} \Big|_7^b = \lim_{b \rightarrow \infty} -b^{-1} - (-7^{-1}) = 7^{-1} = \frac{1}{7}$$

8.4 #7

$$\begin{aligned} \int_1^{\infty} \frac{1}{x^{1.0001}} dx &= \lim_{b \rightarrow \infty} \int_1^b x^{-1.0001} dx = \lim_{b \rightarrow \infty} \frac{x^{-.0001}}{-.0001} \Big|_1^b \\ &= \lim_{b \rightarrow \infty} \frac{x^{-.0001}}{-.0001} \Big|_1^b = \lim_{b \rightarrow \infty} \frac{b^{-.0001}}{-.0001} - \frac{1^{-.0001}}{-.0001} = 0 - \frac{1}{-.0001} = 10000 \end{aligned}$$

8.4 #8

$$\int_1^{\infty} \frac{1}{x^{.999}} dx = \lim_{b \rightarrow \infty} \int_1^b x^{-.999} dx = \lim_{b \rightarrow \infty} x^{.001} \Big|_1^b = \lim_{b \rightarrow \infty} b^{.001} - 1^{.001} \quad \text{DNE}$$

8.4 #14

$$\begin{aligned} \int_0^{\infty} 50e^{-50x} dx &= \lim_{b \rightarrow \infty} \int_0^b 50e^{-50x} dx = \lim_{b \rightarrow \infty} -e^{-50x} \Big|_0^b \\ &= \lim_{b \rightarrow \infty} -e^{-50b} - (-e^{-50 \cdot 0}) = 0 + e^0 = 1 \end{aligned}$$

8.4 #20

$$\int_0^{\infty} \frac{dx}{(4x+1)^3} = \lim_{b \rightarrow \infty} \int_0^b \frac{1}{(4x+1)^3} dx \quad \begin{array}{l} u = 4x+1 \\ du = 4 dx \\ \frac{1}{4} du = dx \end{array}$$

$$= \lim_{b \rightarrow \infty} \int_{x=0}^{x=b} \frac{1}{u^3} \cdot \frac{1}{4} du = \lim_{b \rightarrow \infty} \frac{1}{4} \int_{x=0}^{x=b} u^{-3} du$$

$$= \lim_{b \rightarrow \infty} \frac{1}{4} \left[ \frac{1}{-2} u^{-2} \right]_{x=0}^{x=b} = \lim_{b \rightarrow \infty} \frac{-1}{8} (4x+1)^{-2} \Big|_0^b$$

$$= \lim_{b \rightarrow \infty} \frac{-1}{8} (4b+1)^{-2} - \frac{-1}{8} (4 \cdot 0 + 1)^{-2}$$

$$= 0 + \frac{1}{8} \cdot 1^{-2} = \frac{1}{8}$$