A problem from Börgers 1.1

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This document is meant to show you how to do various simple stuff in LaTeX. If you see anything here that you want to mimic in your writeups, just copy the code and change stuff around as needed.

Exercise: Börgers 1.1 (easy)
In an election involving five voters and three candidates, the preference schedule is:

2	2	1
A	B	C
B	C	A
C	A	B

Determine the set W of winners using (a) the plurality method, (b) the runoff method, (c) the elimination method, (d) Borda count, and (e) the method of pairwise comparison. Is there a Condorcet candidate? ◦

Solution: Part (a)
A and B both got 2 first place votes while C only got 1 first place vote, so A and B both win. Thus $W = \{A, B\}$. □

Solution: Part (b)
Since C got the fewest first place votes, we have a runoff between A and B. Removing C from the election gives 3 voters who favor A over B, and 2 who favor B over A. So A is the winner, so $W = \{A\}$. □

Solution: Part (c)
According to the first place votes, C is the weakest candidate. Thus we eliminate C and count votes for A and B just like in Part (b). Again we have $W = \{A\}$. □

Solution: Part (d)
Since there are three candidates, a first place vote gets 3 points, second place

gets 2 points, and last place gets 1 point. The counts according to the table are:

$$A : 3 \times 2 + 2 \times 1 + 1 \times 2 = 10$$
$$B : 3 \times 2 + 2 \times 2 + 1 \times 1 = 11$$
$$C : 3 \times 1 + 2 \times 2 + 1 \times 2 = 9$$

Thus $W = \{B\}$. □

Solution: Part (e)
When A and B go head to head A wins by 1. When B and C go head to head B wins by 3. When A and C go head to head C wins by 3. So each candidate gets 1 pairwise comparison point, so $W = \{A, B, C\}$. □

Solution: Condorcet?
There is no Condorcet winner, since we just saw that no candidate wins every pairwise comparison. □

Since I love pictures, here's a picture: