

Math 121 HW #5

Section 7.4 # 19, 27, 42,

Section 7.5 # 7, 22

7.4 #19

$$\int_1^2 e^{4u} - \frac{1}{(u+1)^2} du$$

$$= \int_1^2 e^{4u} du - \int_1^2 \frac{1}{(u+1)^2} du$$

$$v = u+1 \\ dv = du$$

$$= \frac{1}{4} e^{4u} \Big|_1^2 - \int_{u=1}^{u=2} \frac{1}{v^2} dv$$

$$= \frac{1}{4} e^{4u} \Big|_1^2 - \left(-v^{-1} \right) \Big|_{u=1}^2$$

$$= \frac{1}{4} e^{4u} \Big|_1^2 + \frac{1}{u+1} \Big|_1^2 = \frac{1}{4} e^8 - \frac{1}{4} e^4 + \frac{1}{3} - \frac{1}{2}$$

7.4 #27

$$\int_0^8 x^{1/3} \sqrt{x^{4/3} + 9} dx$$

$$u = x^{4/3} + 9$$

$$du = \frac{4}{3} x^{1/3} dx$$

$$\frac{3}{4} du = x^{1/3} dx$$

$$= \int_{x=0}^{x=8} \frac{3}{4} \sqrt{u} du = \frac{3}{4} \int_{x=0}^{x=8} u^{1/2} du = \frac{3}{4} \cdot \frac{2}{3} \cdot u^{3/2} \Big|_{x=0}^{x=8}$$

$$= \frac{1}{2} (x^{4/3} + 9)^{3/2} \Big|_0^8 = \frac{1}{2} (8^{4/3} + 9)^{3/2} - \frac{1}{2} (0^{4/3} + 9)^{3/2}$$

$$= \frac{1}{2} (16 + 9)^{3/2} - \frac{1}{2} \cdot 9^{3/2} = \frac{1}{2} (25)^{3/2} - \frac{1}{2} 9^{3/2}$$

$$= \frac{1}{2} \cdot 125 - \frac{1}{2} \cdot 27 = \frac{1}{2} \cdot 98 = 49$$

7.4 #42

$$\text{first part} = \int_{-1}^0 x^2 - 2x \, dx$$

$$= \left. \frac{x^3}{3} - x^2 \right|_{-1}^0 = \frac{0^3}{3} - 0^2 - \left(\frac{(-1)^3}{3} - (-1)^2 \right) = - \left(\frac{-1}{3} - 1 \right) = \frac{4}{3}$$

$$\text{second part} = \int_0^2 x^2 - 2x \, dx = \left. \frac{x^3}{3} - x^2 \right|_0^2 = \frac{8}{3} - 4 = -\frac{4}{3} \quad \text{so area} = \frac{4}{3}$$

$$\text{total area is } \frac{4}{3} + \frac{4}{3} = \boxed{\frac{8}{3}}$$

~~7.4~~ 7.5 #7

Area between $x^2 - 30$ and $10 - 3x$

$$x^2 - 30 = 10 - 3x$$

$$x^2 + 3x - 40 = 0$$

$$(x+8)(x-5) = 0$$

$$x = -8, 5$$

bigger? plug $x=0$

$$x^2 - 30 : -30$$

$$10 - 3x : 10$$

$10 - 3x$ is bigger.

$$\int_{-8}^5 10 - 3x - (x^2 - 30) \, dx$$

$$= \int_{-8}^5 -x^2 - 3x + 40 \, dx$$

$$= \left. \frac{1}{3}x^3 - \frac{3}{2}x^2 + 40x \right|_{-8}^5$$

$$= \frac{1}{3}5^3 - \frac{3}{2}5^2 + 40 \cdot 5 - \left(\frac{1}{3}(-8)^3 - \frac{3}{2}(-8)^2 + 40(-8) \right)$$

$$\approx 366$$

7.5 #22

Area between \sqrt{x} , $x\sqrt{x}$.

$$\sqrt{x} = x\sqrt{x}$$

$$x\sqrt{x} - \sqrt{x} = 0$$

$$\sqrt{x}(x-1) = 0$$

$$x = 0, 1$$

plug $x=1/4$:

$$\sqrt{x} : \sqrt{1/4} = 1/2$$

$$x\sqrt{x} : 1/4 \sqrt{1/4} = 1/4 \cdot 1/2 = 1/8$$

\sqrt{x} is bigger.

$$\int_0^1 \sqrt{x} - x\sqrt{x} \, dx = \int_0^1 x^{1/2} - x^{3/2} \, dx$$

$$= \left. \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} \right|_0^1$$

$$= \frac{2}{3} \cdot 1^{3/2} - \frac{2}{5} \cdot 1^{5/2} - \left(\frac{2}{3} \cdot 0^{3/2} - \frac{2}{5} \cdot 0^{5/2} \right)$$

$$= \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$$