Math 235 HW #1

Section 1.1 #6, 36

Section 1.2 #7, 12, 24

1.1 #6

$\vec{u} = [-1, 3, -2]$

$\vec{v} = [4, 0, -1]$

$\vec{w} = [-3, -1, 2]$

$$\vec{u} + 2(\vec{v} - 4\vec{w}) = \begin{bmatrix}-1\\3\\-2\end{bmatrix} + 2\left(\begin{bmatrix}4\\0\\-1\end{bmatrix} - 4\begin{bmatrix}-3\\-1\\2\end{bmatrix}\right)$$

$$= \begin{bmatrix}-1\\3\\-2\end{bmatrix} + 2\left(\begin{bmatrix}4\\0\\-1\end{bmatrix} + \begin{bmatrix}12\\4\\-8\end{bmatrix}\right)$$

$$= \begin{bmatrix}-1\\3\\-2\end{bmatrix} + 2\begin{bmatrix}16\\4\\-9\end{bmatrix} = \begin{bmatrix}-1\\3\\-2\end{bmatrix} + \begin{bmatrix}32\\8\\-18\end{bmatrix} = \begin{bmatrix}31\\11\\-20\end{bmatrix}$$

1.1 #36

$$x_1\begin{bmatrix}1\\3\end{bmatrix} + x_2\begin{bmatrix}-3\\0\end{bmatrix} + x_3\begin{bmatrix}2\\-4\end{bmatrix} + x_4\begin{bmatrix}0\\5\end{bmatrix} = \begin{bmatrix}-6\\12\end{bmatrix}$$

1.2 #7

$\vec{u} = [-1, 3, 4]$

$$\|\vec{u}\| = \sqrt{1^2 + 3^2 + 4^2} = \sqrt{1 + 9 + 16} = \sqrt{26}$$

So a unit vector parallel to $\vec{u}$ is $\frac{1}{\sqrt{26}}[-1, 3, 4]$

1.2 #12

angle between $\vec{u} = [-1, 3, 4]$
$\vec{v} = [2, 1, -1]$

$\vec{u} \cdot \vec{v} = -2 + 3 - 4 = -3$

$\|\vec{u}\| = \sqrt{26}$

$\|\vec{v}\| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$-3 = \sqrt{26} \cdot \sqrt{6} \cdot \cos\theta$$

$$\cos\theta = \frac{-3}{\sqrt{26}\sqrt{6}}$$

$$\theta = \cos^{-1}\left(\frac{-3}{\sqrt{26}\sqrt{6}}\right) = 1.813 \text{ rad.}$$

$$= 103 \text{ deg.}$$

1.2 #24 The angle between two unit vectors $\vec{u}_1, \vec{u}_2$ is $\arccos(\vec{u}_1 \cdot \vec{u}_2)$.

Pf We know $\vec{u}_1 \cdot \vec{u}_2 = \|\vec{u}_1\| \, \|\vec{u}_2\| \cos\theta$,

but $\|\vec{u}_1\| = \|\vec{u}_2\| = 1$ since they're unit vectors.

So $\vec{u}_1 \cdot \vec{u}_2 = \cos\theta$

so $\theta = \cos^{-1}(\vec{u}_1 \cdot \vec{u}_2)$ as desired.