Math 235 HW # 11

Section 5.1 # 3, 19, 23 abc, 24, 26

5.1 #3

$$A=\begin{bmatrix}-1 & -2\\ 4 & 5\end{bmatrix} \qquad |\lambda I - A| = \begin{vmatrix}\lambda+1 & 2\\ -4 & \lambda-5\end{vmatrix} = (\lambda+1)(\lambda-5)+8$$

$$= \lambda^2-4\lambda-5+8 = \lambda^2-4\lambda+3$$

$$= (\lambda-3)(\lambda-1)$$

$\lambda = 3, 1$

$\lambda=3$

$$\left[\begin{array}{cc|c}4 & 2 & 0\\ -4 & -2 & 0\end{array}\right] \to \left[\begin{array}{cc|c}1 & 1/2 & 0\\ 0 & 0 & 0\end{array}\right] \quad \begin{array}{l} y=r\\ x+\tfrac12 r=0\\ x=-1/2\, r\end{array} \quad \begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-1/2\, r\\ r\end{bmatrix}=r\begin{bmatrix}-1/2\\1\end{bmatrix}$$

$\begin{bmatrix}-1/2\\1\end{bmatrix}$ is an eigenvector with eigenvalue 3.

$\lambda=1$

$$\left[\begin{array}{cc|c}2 & 2 & 0\\ -4 & -4 & 0\end{array}\right] \to \left[\begin{array}{cc|c}1 & 1 & 0\\ 0 & 0 & 0\end{array}\right] \quad \begin{array}{l} y=r\\ x=-r\end{array} \quad \begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-r\\ r\end{bmatrix}=r\begin{bmatrix}-1\\1\end{bmatrix}$$

$\begin{bmatrix}-1\\1\end{bmatrix}$ is an eigenvector with eigenvalue 1.

5.1 #19 $\quad T([x_1, x_2, x_3]) = [x_1+x_3, x_2, x_1+x_3]$

matrix: $A=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}$

$$|\lambda I - A| = \begin{vmatrix}\lambda-1 & 0 & -1\\ 0 & \lambda-1 & 0\\ -1 & 0 & \lambda-1\end{vmatrix} = (\lambda-1)\begin{vmatrix}\lambda-1 & -1\\ -1 & \lambda-1\end{vmatrix}$$

$$= (\lambda-1)\left((\lambda-1)^2-1\right) = (\lambda-1)(\lambda^2-2\lambda+\cancel{1}\cancel{-1})$$

$$= (\lambda-1)(\lambda-2)\lambda. \qquad \lambda = 0, 1, 2.$$

$\lambda=0$

$$\left[\begin{array}{ccc|c}-1 & 0 & -1 & 0\\ 0 & -1 & 0 & 0\\ -1 & 0 & -1 & 0\end{array}\right] \to \left[\begin{array}{ccc|c}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\end{array}\right] \quad \begin{array}{l} z=r\\ y=0\\ x=-r\end{array} \quad \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-r\\0\\r\end{bmatrix}=r\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

$[-1, 0, 1]$ is an eigenvector with eigval 0

$\lambda=1$

$$\left[\begin{array}{ccc|c}0 & 0 & -1 & 0\\ 0 & 0 & 0 & 0\\ -1 & 0 & 0 & 0\end{array}\right] \to \left[\begin{array}{ccc|c}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\end{array}\right] \quad \begin{array}{l} z=0\\ y=r\\ x=0\end{array} \quad \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\r\\0\end{bmatrix}=r\begin{bmatrix}0\\1\\0\end{bmatrix}$$

$[0, 1, 0]$ is an eigvec with eigval 1.

5.1 #19 ctd

$\lambda = 2$ $\quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\quad \begin{array}{l} z = r \\ y = 0 \\ x = r \end{array}$ $\quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r \\ 0 \\ r \end{bmatrix} = r\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$[1,0,1]$ is an eigvec with eigval 2.

5.1 # 23a False - when you solve the char. polyn, the solutions might not be real. e.g. $\lambda^2+1=0 \to \lambda = \pm i$

23b False - the roots of the char polyn might not be distinct. e.g. $(\lambda-1)^2=0 \to \lambda = 1, 1$

23c True - the roots of the char polyn may be complex, and might not be distinct.

5.1 #24 Let T be linear, $\lambda \in \mathbb{R}$, $S = \{\vec{v} \mid T(\vec{v}) = \lambda\vec{v}\}$. Show S is a subspace.

Pf closed under + Take $\vec{u}, \vec{v} \in S$, so $T(\vec{u}) = \lambda\vec{u}$ and $T(\vec{v}) = \lambda\vec{v}$.

We'll show $\vec{u}+\vec{v} \in S$, i.e. $T(\vec{u}+\vec{v}) = \lambda(\vec{u}+\vec{v})$.

Here goes: $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$ (T is linear)

$= \lambda\vec{u} + \lambda\vec{v} = \lambda(\vec{u}+\vec{v})$ ✓

closed under smult Take $\vec{u} \in S$, $k \in \mathbb{R}$. So $T(\vec{u}) = \lambda\vec{u}$,

will show $k\vec{u} \in S$, i.e. $T(k\vec{u}) = \lambda k\vec{u}$.

We have: $T(k\vec{u}) = kT(\vec{u})$ (T is linear)

$= k\lambda\vec{u} = \lambda k\vec{u}$ ✓

5.1 #26 $T(f) = f^{(4)}$ fourth deriv.

$T(e^{ax}) = a^4 e^{ax}$ so e^{ax} is an eigvec with eigval a^4

$T(e^{-ax}) = (-a)^4 e^{-ax}$
$= a^4 e^{-ax}$ so e^{-ax} " — " — — —

$T(\sin ax) = a^4 \sin ax$ so $\sin ax$ — — — —

$T(\cos ax) = a^4 \cos ax$ so $\cos ax$ — — — —