Math 235 HW #3

Section 1.4: 4, 9, 17, 25, 43

#4

$$\begin{bmatrix} 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 2 \\ 1 & 3 & 2 & -4 \end{bmatrix} \xrightarrow[R1 \to R3]{} \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & -2 \end{bmatrix} \xrightarrow[\substack{R3-3R2 \\ \to R3}]{} \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -8 \end{bmatrix}$$

this is row echelon form.

$$\xrightarrow[R3/-8]{} \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[\substack{R2-2R3 \\ \to R2}]{\substack{R1+4R3 \\ \to R1}} \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R1-2R2} \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ Reduced Row Echelon Form.

#9

$$\begin{array}{c} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \\ \begin{array}{cccc} & & s & r \end{array} \end{array}$$

$x_4 = r$

$x_3 = s$

R2: $x_2 + s + 3r = -2$

$x_2 = -2 - s - 3r$

R1: $x_1 + 2s = 1$

$x_1 = 1 - 2s$

So $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1-2s \\ -2-s-3r \\ s \\ r \end{bmatrix}$$

when $x_3 = 3$, $x_4 = -2$, $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \\ 3 \\ -2 \end{bmatrix}$$

#17

$$\left[\begin{array}{cc|c} 1 & -2 & 3 \\ 3 & -1 & 14 \\ 1 & -7 & -2 \end{array}\right] \xrightarrow[\substack{R3-R1 \\ \to R3}]{\substack{R2-3R1 \\ \to R2}} \left[\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 5 & 5 \\ 0 & -5 & -5 \end{array}\right] \xrightarrow[R2/5]{} \left[\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 1 & 1 \\ 0 & -5 & -5 \end{array}\right]$$

$$\xrightarrow[\substack{R3+5R2 \\ \to R3}]{} \left[\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

Back-subst: $y = 1$

$x - 2y = 3$

$x = 5$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

#25 is $\vec{b} = \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}$ in span of $\begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ 5 \end{bmatrix}$?

This is equiv. to: is there a soln to

$$\begin{bmatrix} 0 & 1 & -3 \\ 2 & 4 & -1 \\ 4 & -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix} \; ?$$

$$\left[\begin{array}{ccc|c} 0 & 1 & -3 & 3 \\ 2 & 4 & -1 & 5 \\ 4 & -2 & 5 & 3 \end{array}\right] \xrightarrow{R1 \leftrightarrow R2} \left[\begin{array}{ccc|c} 2 & 4 & -1 & 5 \\ 0 & 1 & -3 & 3 \\ 4 & -2 & 5 & 3 \end{array}\right] \xrightarrow{R1/2} \left[\begin{array}{ccc|c} 1 & 2 & -1/2 & 5/2 \\ 0 & 1 & -3 & 3 \\ 4 & -2 & 5 & 3 \end{array}\right]$$

$$\xrightarrow[\to R3]{R3-4R1} \left[\begin{array}{ccc|c} 1 & 2 & -1/2 & 5/2 \\ 0 & 1 & -3 & 3 \\ 0 & -10 & 7 & -7 \end{array}\right] \xrightarrow{R2/-10} \left[\begin{array}{ccc|c} 1 & 2 & 1/2 & 5/2 \\ 0 & 1 & -3 & 3 \\ 0 & 1 & -.7 & +.7 \end{array}\right]$$

$$\xrightarrow[\to R3]{R3-R2} \left[\begin{array}{ccc|c} 1 & 2 & .5 & 2.5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 2.3 & -2.3 \end{array}\right]$$

There will be a solution, (doesn't matter what) so $\vec{b}$ <u>is</u> in the span.

#43 Find E s.t.

$$E \begin{bmatrix} 1 & 3 & 1 & 4 \\ 0 & 1 & 2 & 1 \\ 3 & 4 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 & 4 \\ 2 & 7 & 4 & 9 \\ 3 & 4 & 5 & 1 \end{bmatrix}$$

The effect of E is to do $R2 + 2 \cdot R1 \to R2$, so

E is the elementary matrix: $E = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$