Math 235 HW #4

Section 1.4 #56

1.5 #6a, ~~#~~ 14, 17, 23 ab

1.4 #56 $y = ax^2 + bx + c$ goes thru $(1,4)$, $(-1,0)$, and $(2,3)$.

Then $-4 = a \cdot 1^2 + b \cdot 1 + c$ (plug $x=1$, you get $y=-4$)

so $-4 = a + b + c$

also $0 = a - b + c$

$3 = 4a + 2b + c$

So $\left[\begin{array}{ccc|c} 1 & 1 & 1 & -4 \\ 1 & -1 & 1 & 0 \\ 4 & 2 & 1 & 3 \end{array}\right] \to \text{row ops} \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -5 \end{array}\right]$

So $a = 3$
$b = -2$
$c = -5$

1.5 #6a

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 0 & 1 & 0 \\ -3 & 1 & -7 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 5 & -2 & 1 & 0 \\ 0 & 4 & -10 & 3 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2.5 & 1 & -.5 & 0 \\ 0 & 4 & -10 & 3 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 1.5 & 0 & .5 & 0 \\ 0 & 1 & -2.5 & 1 & -.5 & 0 \\ 0 & 0 & 0 & -1 & 2 & 1 \end{array}\right]$$

not invertible!

1.5 #14 solve

$$2x_1 + x_2 + 4x_3 = 5$$
$$3x_1 + 2x_2 + 5x_3 = 3$$
$$-x_2 + x_3 = 8$$

with an inverse.

First we do $\left[\begin{array}{ccc|ccc} 2 & 1 & 4 & 1 & 0 & 0 \\ 3 & 2 & 5 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \to \dots \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7 & 5 & 3 \\ 0 & 1 & 0 & 3 & -2 & -2 \\ 0 & 0 & 1 & 3 & -2 & -1 \end{array}\right]$

So $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & -1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -7 & 5 & 3 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \end{bmatrix}$.

then our solution is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -7 & 5 & 3 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}$

1.5 #17

$A^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}$. Find C such that $ACA = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 2 \\ 2 & 1 & 4 \end{bmatrix}$

Because inverses cancel, we have $A^{-1}(ACA)A^{-1} = C$.

so $C = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 2 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 6 & 11 \\ -1 & 7 & 10 \\ 11 & 8 & 22 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 46 & 33 & 30 \\ 39 & 29 & 26 \\ 99 & 68 & 63 \end{bmatrix}$

1.5 23 ab

a "If $AC = BC$ and C is invertible, then $A = B$"

True, since you can multiply both sides by C^{-1} on the right: $AC = BC$ so $ACC^{-1} = BCC^{-1}$ so $A = B$.

b "If $AB = O$ and B is invertible, then $A = O$"

True for the same reason:

$AB = O$ so $ABB^{-1} = OB^{-1}$

so $A = O$ (since $OB^{-1} = O$)