1.6   2, #, 8, 16, 23, 26

1.6 #2 $\{[x, x+1] \mid x \in \mathbb{R}\}$

closed under + $[a, a+1] + [b, b+1] = [a+b, a+b+2]$

↗ not 1

not closed under +,

so **not** a subspace.

1.6 #8 $\{[2x, x+y, y] \mid x, y \in \mathbb{R}\}$

add: $[2x, x+y, y] + [2a, a+b, b] = [2(x+a), x+a+y+b, y+b]$

it **is** closed under +.

Smult: $k[2x, x+y, y] = [2kx, kx+ky, ky]$

it **is** closed under smult,

It is a subspace

1.6 #16

$$x - y = 0$$
$$2x - 2y = 0$$

$\begin{bmatrix} 1 & -1 & | & 0 \\ 2 & -2 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$

↑ r

$y = r$
$x = r$

$\begin{bmatrix} x \\ y \end{bmatrix} = r \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ so $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ is a basis for the solutions.

1.6 #23 ~~Basis~~ $\text{sp}\left\{\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}\right\}$

$$\begin{bmatrix} -1 & 2 & | & 0 \\ 3 & 1 & | & 0 \\ 1 & 4 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & | & 0 \\ 3 & 1 & | & 0 \\ 1 & 4 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & | & 0 \\ 0 & 7 & | & 0 \\ 0 & 6 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

they are indep, so they __are__ a basis for the span.

1.6 #26 $\text{sp}\left\{\begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix}\right\}$

$$\begin{bmatrix} 2 & 2 & 3 & | & 0 \\ 1 & -3 & 2 & | & 0 \\ 0 & 1 & 6 & | & 0 \\ 2 & 0 & 6 & | & 0 \end{bmatrix} \rightarrow \cdots \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

they are indep, so they __are__ a basis for the span.