Math 235 HW # 7

Section 2.3 #9, 14, 18, 31

Section 2.4 #4

2.3 #9

$T([-1,2]) = [1,0,0]$

$T([2,1]) = [0,1,2]$.

Let's find $T([0,10])$. First write $[0,10] = c_1[-1,2] + c_2[2,1]$

$-c_1 + 2c_2 = 0$
$2c_1 + c_2 = 10$

$$\left[\begin{array}{cc|c} -1 & 2 & 0 \\ 2 & 1 & 10 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} c_1 = 4 \\ c_2 = 2 \end{array}$$

So $T([0,10]) = T(4[-1,2] + 2[2,1])$

$= 4T([-1,2]) + 2T([2,1])$

$= 4[1,0,0] + 2[0,1,2]$

$= [4,2,4]$

2.3 #14

$T([x_1,x_2]) = [2x_1 - x_2,\ x_1 + x_2,\ x_1 + 3x_2]$

$T([1,0]) = [2,1,1]$

$T([0,1]) = [-1,1,3]$

the matrix is $\begin{bmatrix} 2 & -1 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$

2.3 #18 $T([x_1, x_2, x_3]) = x_1 + x_2 + x_3$

$T([1,0,0]) = 1$

$T([0,1,0]) = 1$ matrix is $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

$T([0,0,1]) = 1$

2.3 #31 If T and T' are linear, show $T' \circ T$ is linear.

Preserves + We need to show $T' \circ T(\vec{u} + \vec{v}) = T' \circ T(\vec{u}) + T' \circ T(\vec{v})$

$T' \circ T(\vec{u} + \vec{v}) = T'(T(\vec{u} + \vec{v})) = T'(T(\vec{u}) + T(\vec{v}))$ since T is linear

$= T'(T(\vec{u})) + T'(T(\vec{v}))$ since T' is linear

$= T' \circ T(\vec{u}) + T' \circ T(\vec{v})$ as desired.

Preserves smult We need $T' \circ T(r\vec{u}) = r\, T' \circ T(\vec{u})$

$T' \circ T(r\vec{u}) = T'(T(r\vec{u})) = T'(r\, T(\vec{u}))$ since T is linear

$= r\, T'(T(\vec{u}))$ since T' is linear

2.4 #4 Formulas for $\sin 3\theta$, $\cos 3\theta$.

rotation by 3θ is $\begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$

this equals 3 rotations by θ: $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta\ \sin^2\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^3\theta - \cos\theta\sin^2\theta - 2\sin^2\theta\cos\theta & \sim \\ 2\sin\theta\cos^2\theta + \cos^2\theta\sin^3\theta & \sim \end{bmatrix}$$

So $\cos 3\theta = \cos^3\theta - \cos\theta\sin^2\theta - 2\sin^2\theta\cos\theta$

$\sin 3\theta = 2\sin\theta\cos^2\theta + \cos^2\theta\sin^3\theta$