Math ~~32~~235 HW #8

Section 3.1 # 1, 10, 14

Section 3.2 # 12, 20

3.1 #1 $\mathbb{R}^2$ with usual addition and $r[x,y] = [ry, rx]$.

We need to check the 8 axioms - A1 - A4 are true since it's the usual +

But S3 and S4 are false. (We only need to show one is false)

S4 is easiest: $1[x,y] = [y,x] \neq [x,y]$

So it's <u>not</u> a vectorspace.

3.1 #10 The set of 2×2 matrices like $\begin{bmatrix} * & 1 \\ 1 & * \end{bmatrix}$ with usual operations.

Since it's the usual operations, the 8 axioms are true.

We'll think about closure.

It's <u>not</u> closed under addition:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

these are both in the set, but the sum isn't.

So it's <u>not</u> closed under addition.

So <u>not</u> a vectorspace.

3.1 #14 $\mathbb{Q}$ is not closed under scalar mult, since the scalar can be irrational.

For example $\underset{\text{in } \mathbb{R}}{\pi} \cdot \underset{\text{in } \mathbb{Q}}{1} = \underset{\text{not in } \mathbb{Q}.}{\pi}$

3.2 #12

Is it indep? $\{1, 4x+3, 3x-4, x^2+2, x-x^2\}$

$$c_1 \cdot 1 + c_2(4x+3) + c_3(3x-4) + c_4(x^2+2) + c_5(x-x^2) = 0$$

$$(c_4-c_5)x^2 + (4c_2+3c_3+c_5)x + c_1+3c_2-4c_3+2c_4 = 0$$

$$\begin{aligned} c_4 - c_5 &= 0 \\ 4c_2+3c_3+c_5 &= 0 \\ c_1+3c_2-4c_3+2c_4 &= 0 \end{aligned} \rightarrow \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & -4 & 2 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & -\frac{25}{4} & 0 & \frac{5}{4} & 0 \\ 0 & 1 & 3/4 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

we'll get ~~some~~ free variables, so not all c_i must be 0,

so not independent.

3.2 #20

Is it a basis for P_2? $\{x, x^2+1, (x-1)^2\}$

P_2 has dim 3, so this will be a basis as long as it's indep.

$$c_1 x + c_2(x^2+1) + c_3(x-1)^2 = 0$$

$$c_1 x + c_2(x^2+1) + c_3(x^2-2x+1) = 0$$

$$(c_2+c_3)x^2 + (c_1-2c_3)x + c_2+c_3 = 0$$

$$\begin{aligned} c_2+c_3 &= 0 \\ c_1-2c_3 &= 0 \\ c_2+c_3 &= 0 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

not indep, so it's not a basis.