Math 235 HW #9

Section ~~#~~ 3.3 #4, 10

Section 3.4 #1, 8ab, 21

3.3 #4

$$c_1\begin{bmatrix}0\\1\\1\end{bmatrix} + c_2\begin{bmatrix}2\\0\\0\end{bmatrix} + c_3\begin{bmatrix}0\\3\\0\end{bmatrix} = \cancel{\begin{bmatrix}4\\6\\2\end{bmatrix}}\begin{bmatrix}4\\-2\\1\end{bmatrix}$$

$$\cancel{\left[\begin{array}{ccc|c}2&0&0&4\\0&1&0&6\\0&1&1&2\end{array}\right] \to \left[\begin{array}{ccc|c}1&0&0&2\\0&1&0&6\\0&0&1&-4\end{array}\right]} \quad \left[\begin{array}{ccc|c}0&2&0&4\\1&0&3&-2\\1&0&0&1\end{array}\right] \to \left[\begin{array}{ccc|c}1&&&1\\&1&&2\\&&1&-1\end{array}\right]$$

coords are ~~[...]~~ $[1, 2, -1]$

3.3 #10

coords of $\begin{bmatrix}1&-2\\3&4\end{bmatrix}$ wrt $\left\{\begin{bmatrix}0&1\\1&0\end{bmatrix}, \begin{bmatrix}0&-1\\0&0\end{bmatrix}, \begin{bmatrix}1&-1\\0&3\end{bmatrix}, \begin{bmatrix}0&1\\0&1\end{bmatrix}\right\}$

$$\begin{bmatrix}1&-2\\3&4\end{bmatrix} = c_1\begin{bmatrix}0&1\\1&0\end{bmatrix} + c_2\begin{bmatrix}0&-1\\0&0\end{bmatrix} + c_3\begin{bmatrix}1&-1\\0&3\end{bmatrix} + c_4\begin{bmatrix}0&1\\0&1\end{bmatrix}$$

$$= \begin{bmatrix}c_3 & c_1-c_2-c_3+c_4\\ c_1 & 3c_3+c_4\end{bmatrix}$$

$c_3 = 1$

$c_1 - c_2 - c_3 + c_4 = -2$

$c_1 = 3$

$3c_3 + c_4 = 4$

$$\left[\begin{array}{cccc|c}0&0&1&0&1\\1&-1&-1&1&-2\\1&0&0&0&3\\0&0&3&1&4\end{array}\right] \to \left[\begin{array}{cccc|c}1&0&0&0&3\\0&1&-1&1&-5\\0&0&1&0&1\\0&0&3&1&4\end{array}\right]$$

$$\to \left[\begin{array}{cccc|c}1&&&&3\\&1&&&5\\&&1&&1\\&&&1&1\end{array}\right]$$

so the coords are $[3, 5, 1, 1]$

3.4 #1 $\quad T: F \to \mathbb{R} \qquad T(f) = f(-4)$

$$T(f+g) = (f+g)(-4) = f(-4) + g(-4)$$

$$T(f) + T(g) = f(-4) + g(-4)$$

✓ pres +

$$T(kf) = (kf)(-4) = kf(-4)$$

$$kT(f) = kf(-4)$$

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it <u>is</u> linear.

3.4 #8 $\quad T(e^{2x}) = x^2, \quad T(e^{3x}) = \sin x, \quad T(1) = \cos 5x,$ T is linear.

<u>a</u> $\quad T(e^{5x}) = ?$ $\quad$ not enough info since e^{5x} is not a lin. combo of $e^{2x}, e^{3x}, 1$.

(Don't say $T(e^{5x}) = T(e^{3x} \cdot e^{2x}) = T(e^{3x}) \cdot T(e^{2x})$, since T does not preserve multiplication of functions)

<u>b</u> $\quad T(3 + 5e^{3x}) = 3T(1) + 5T(e^{3x})$

$$= 3 \cdot \cos 5x + 5 \cdot \sin x$$

3.4 #21

$T(x^3) = 3x^2 \to [0, 3, 0, 0]$

$T(x^2) = 2x \to [0, 0, 2, 0]$

$T(x) = 1 \to [0, 0, 0, 1]$

$T(1) = 0 \to [0, 0, 0, 0]$

so $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

f. $\quad T(4x^3 - 5x^2 + 10x - 13)$, $\quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ -5 \\ 10 \\ -13 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ -10 \\ 10 \end{bmatrix}$

so $T(4x^3 - 5x^2 + 10x - 13)$

$$= 12x^2 - 10x + 10$$