# Math 119 HW #10

Section 6.1 #11, 15, 19, 25, 27

**#11** $f(x) = x^3 - 6x^2 + 9x - 8$ on $[0, 5]$

$f'(x) = 3x^2 - 12x + 9$

$3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1, 3$

| $x$ | $f(x)$ |
|---|---|
| 0 | $-8$ |
| 5 | 12 |
| 1 | $-4$ |
| 3 | $-8$ |

So 12 is the max. at $x = 5$

$-8$ is the min. at $x = 0$ or $x = 3$

**#15** $f(x) = x^4 - 18x^2 + 1$ on $[-4, 4]$

$f'(x) = 4x^3 - 36x$

$4x^3 - 36x = 0$

$x^3 - 9x = 0$

$x(x^2 - 9) = 0$

$x(x+3)(x-3) = 0$

$x = 0, 3, -3$

| $x$ | $f(x)$ |
|---|---|
| 0 | 1 |
| 3 | $-80$ |
| $-3$ | $-80$ |
| $-4$ | $-31$ |
| 4 | $-31$ |

1 is the max. at $x = 0$

$-80$ is the min. at $x = \pm 3$

**#19** $f(x) = \dfrac{x-1}{x^2+1}$ on $[1, 5]$

$f'(x) = \dfrac{(x^2+1)\cdot 1 - (x-1)\cdot 2x}{(x^2+1)^2}$

$= \dfrac{x^2 + 1 - 2x^2 + 2x}{(x^2+1)^2}$

$= \dfrac{-x^2 + 2x + 1}{(x^2+1)^2}$

$f' = 0$: $\dfrac{-x^2 + 2x + 1}{(x^2+1)^2} = 0$

$-x^2 + 2x + 1 = 0$

quad. form:

$x = \dfrac{-2 \pm \sqrt{4 - 4 \cdot -1}}{-2}$

$= \dfrac{-2 \pm \sqrt{8}}{-2}$

$= \dfrac{-2 \pm 2\sqrt{2}}{-2}$

$= -1 \pm \sqrt{2}$

| $x$ | $f(x)$ |
|---|---|
| 1 | 0 |
| 5 | .15 |
| $1+\sqrt{2}$ | .207 |
| ~~$1-\sqrt{2}$~~ | |

outside of $[1, 5]$

max is .207 at $x = 1 + \sqrt{2}$

min is 0 at $x = 1$

**#25** $f(x) = x^2 - 8\ln x$ on $[1,4]$

$f'(x) = 2x - \frac{8}{x}$

$f'$ DNE at $x=0$

$f'=0$ at:

$2x - \frac{8}{x} = 0$

$2x^2 - 8 = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

| $x$ | $f(x)$ |
|---|---|
| 1 | 1 |
| 4 | 4.91 |
| ~~0~~ (outside the interval) | |
| ~~-2~~ (outside the interval) | |
| 2 | -1.5 |

max is 4.91 at $x=4$

min is $-1.5$ at $x=2$

**#27** $f(x) = x + e^{-3x}$ on $[-1,3]$

$f'(x) = 1 + -3e^{-3x}$

$0 = 1 - 3e^{-3x}$

$3e^{-3x} = 1$

$e^{-3x} = \frac{1}{3}$

$-3x = \ln(\frac{1}{3})$

$x = \frac{\ln(\frac{1}{3})}{3}$

$x \simeq .366$

| $x$ | $f(x)$ |
|---|---|
| -1 | 19.1 |
| 3 | 3.0001 |
| .366 | .7 |

19.1 is the max, at $x = -1$

.7 is the min, at $x \simeq .366$