Math 119 HW #8

Section 5.2 # 26

Section 5.3 # 7, 34, 39, 60

5.2 #26 $\quad f(x) = x^2 + \frac{1}{x} = x^2 + x^{-1}$

$f'(x) = 2x + -x^{-2} = 2x - \frac{1}{x^2}$

$f'(x) = 0$

$2x - \frac{1}{x^2} = 0$

$2x = \frac{1}{x^2}$

$2x^3 = 1$

$x^3 = 1/2$

$x = \sqrt[3]{1/2} = .79$

$f'(x)$ DNE $\quad x^2 = 0$

$x = 0$

crit pts at $x=0$, $x = \sqrt[3]{1/2} \approx .79$

$f(0)$ DNE, so $x=0$ is not an extremum - it's an asymptote.

x		0		$.79$	
$f'(x)$		DNE	$-$	0	$+$

$f'(.5) = 2 \cdot .5 - \frac{1}{.5^2}$

$= 1 - \frac{1}{.25} = 1 - 4 = -3$

$f'(1) = 2 \cdot 1 - \frac{1}{1^2} = 2 - 1 = 1$

So $x = .79$ is a rel. min.

5.3 #7 $\quad f(x) = \frac{x^2}{1+x} \qquad f'(x) = \frac{(1+x)\cdot 2x - x^2 \cdot 1}{(1+x)^2} = \frac{2x + 2x^2 - x^2}{(1+x)^2}$

$= \frac{2x + x^2}{(1+x)^2}$

$f''(x) = \frac{(1+x)^2 \cdot (2+2x) - (2x+x^2)\cdot 2(1+x)^1 \cdot 1}{(1+x)^4}$

$f''(0) = \frac{(1+0)^2 \cdot (2+2\cdot 0) - (2\cdot 0 + 0^2)\cdot 2(1+0)}{(1+0)^4} = 2$

$f''(2) = \frac{(1+2)^2 (2+2\cdot 2) - (2\cdot 2 + 2^2)\cdot 2(1+2)}{(1+2)^4} = \frac{2}{27}$

5.3 #34

$f(x) = 8 - 6x - x^2$

$f'(x) = -6 - 2x$

$f''(x) = -2$

f'' is always negative, so

f is concave down on $(-\infty, \infty)$,

concave up never,

no inflection pts.

5.3 #39

$f(x) = x(x+5)^2$

$f'(x) = x \cdot 2(x+5) + (x+5)^2 \cdot 1$

$= 2x^2 + 10x + (x+5)^2$

$f''(x) = 4x + 10 + 2(x+5) = 4x + 10 + 2x + 10 = 6x + 20$

$f'' = 0$

$6x + 20 = 0$

$6x = -20$

$x = -\frac{20}{6} = 3.33$

-3.33

$f''(x)$ $-$ 0 $+$

$f''(0) = 20$

$f''(-4) = 6 \cdot -4 + 20$
$= -4$

f is concave down on $(-\infty, -\frac{20}{6})$

Concave up on $(-\frac{20}{6}, \infty)$

$x = -\frac{20}{6}$ is an inflection pt.

5.3 #60

$f(x) = 2x^3 - 4x^2 + 2$

$f'(x) = 6x^2 - 8x$

$f''(x) = 12x - 8$

$f'(x) = 0$

$6x^2 - 8x = 0$

$x(3x - 4) = 0$

$x = 0$ or $3x - 4 = 0$

$x = 4/3$

crit pts at $x = 0$, $x = 4/3$

$f''(0) = 12 \cdot 0 - 8 = -8$ so $x = 0$ is a local max

$f''(4/3) = 12 \cdot \frac{4}{3} - 8 = 4 \cdot 4 - 8$ pos so $x = 4/3$ is a local min