Math 300 HW #11

Section 11.2 #23a, 29, 45

11.3 #12, 38

11.2 #23a WTS: If $x>1$, $\left|\frac{1}{5}x^2-42x-8\right| \le 51\,|x^2|$

We have:

$$\left|\tfrac{1}{5}x^2-42x-8\right| \le \left|\tfrac{1}{5}x^2\right| + |42x| + |8|$$

$$\le |x^2| + 42|x^2| + 8|x^2|$$

$$= 51|x^2| \quad \text{as desired.}$$

11.2 #29 WTS $\frac{1}{5}x^2-42x-8$ is $\Omega(x^2)$

let $x > \frac{2}{1/5}(42+8) = 500$

So $\frac{1/5}{2}x > 42+8 > 42+8\cdot\frac{1}{x}$

multiply by x: $\frac{1/5}{2}x^2 > 42x+8$

so $\frac{1}{5}x^2 - \frac{1/5}{2}x^2 > 42x+8$,

$$\tfrac{1}{5}x^2-42x-8 > \frac{\frac{1}{5}}{2}x^2 = \tfrac{1}{10}x^2$$

since $x>500$, both sides above are positive, so

$$\left|\tfrac{1}{5}x^2-42x-8\right| > \tfrac{1}{10}|x^2|$$

so $\frac{1}{5}x^2-42x-8$ is $\Omega(x^2)$

with $A=\frac{1}{10}$, $a=500$

11.2 #45

$$\sum_{k=1}^{n}(k+3) = \sum_{k=1}^{n} k + \sum_{k=1}^{n} 3$$

$$= \frac{n(n+1)}{2} + 3n \quad \text{is} \quad \Theta(n^2)$$

11.3 #12

when $k=1$, we do the inner loop $n-(1+1)+1 = n-1$ times

when $k=2$, — — — — — $n-(2+1)+1 = n-2$ times

when $k=3$, — — — — — — — — — $n-3$ times

⋮ — — — — 1 times

when $k=n-1$,

So the total # of comparisons is the sum:

$(n-1)+(n-2)+\dots+1$

write it backwards: $1+2+\dots+(n-1) = \frac{(n-1)(n-1+1)}{2}$ is $\Theta(n^2)$.

11.3 #38

We have 1 addition each time for the outer for loop, which runs n times.

AND, 1 mult. for each time in the inner loop.

when $i=1$, this is 1 time,

$i=2$ — — 2 times,

⋮

$i=n$ n times.

So the # of mults is $1+2+\dots+n = \frac{n(n+1)}{2}$

So the total # of operations is $S_n = n + \frac{n(n+1)}{2}$