MA 300 HW #2

Section 9.4 6a, 16, 35

Section 9.5 #6&d, 15c

9.4 #6a Given any set of 7 integers, there must be 2 who have the same remainder when divided by 6 are equal.

Pf The possible remainders are $\{0,1,\dots,5\}$
let A be the set with 7 integers, and let

$f: A \to \{0,\dots,5\}$ be the remainder when dividing by 6.

Then f is a function from 7 elements to 6, so it's not 1 to 1, so two elements of A have the same remainder.

9.4 #16 Of the integers 1 to 100, 20 are divisible by 5.
Thus we must choose 81 in order to be sure of choosing one div. by 5.

9.4 #35 let A be the set of 52 integers, let

$f: A \to \{00, 50, \{01,99\}, \{02,98\}, \dots, \{49,51\}\}$ be

the map which takes x to the element including the last 2 digits of x.

f maps a set of 52 elements to a set of 51 elements so it cannot be 1 to 1, so two numbers say x & y have $f(x)=f(y)$.

If $f(x)=f(y)=00$ or $f(x)=f(y)=50$, then $x+y$ ends in 00 and thus $x+y$ is divisible by 100.

If $f(x)=f(y)=\{a,b\}$, then either x & y both end in the same 2 digits, or the last 2 digits sum to 100. When x & y end in the same 2 digits we have $x-y$ div. by 100. When the last digits sum to 100, we have $x+y$ divisible by 100.

9.5 #6cd

c: Say A & B only go together.

using A&B : $\binom{13}{4}$ need 4 more from the remaining people

without A&B : $\binom{13}{6}$ need 6 from the remaining

So it's $\binom{13}{4}+\binom{13}{6}$

d: i 3 men : $\binom{8}{3}$

3 women : $\binom{7}{3}$ $\Big\}$ $\binom{8}{3}\binom{7}{3}$

ii We'll count the # with no women, then subtract from the total #.

with no women : $\binom{8}{6}$

total # : $\binom{15}{6}$

So it's $\binom{15}{6}-\binom{8}{6}$

9.5 #15c There's 50 odds & 50 evens.

To sum to even, it must be even + even or odd + odd.

So we need the # of ways to choose 2 evens or 2 odds,

So it's $\binom{50}{2}+\binom{50}{2}$.