

Math 300 HW ~~#2~~ #3

Section 9.6 #3, 13, 19a

Section 9.7 #7, 12

9.6 #3 a $\binom{20+6-1}{20} = \binom{25}{20}$ 20-comb w/rep from 6

b 3 eclairs dictated, then 17-comb from 6:

$$\binom{17+6-1}{17} = \binom{22}{17}$$

c n eclairs + 1 éclair + 2 eclairs

= 20-comb w/rep from 5 + 19 comb w/rep from 5 + 18 comb w/rep from 5

$$= \binom{20+5-1}{20} + \binom{19+5-1}{19} + \binom{18+5-1}{18}$$

$$= \binom{24}{20} + \binom{23}{19} + \binom{22}{18}$$

Silly under w/

or just subtract:

$$\binom{25}{20} - \binom{22}{17}$$

9.6 #13

$$y_1 + y_2 + y_3 + y_4 = 30, \quad y_i \geq 2, \quad y_i \in \mathbb{Z}.$$

8 assignments are forced, so it remains to distribute 22 into 4,

so it's 22 comb w/rep from 4, which is

$$\binom{22+4-1}{22} = \binom{25}{22}$$

9.6 #19a

can use at most 10 eclairs.

Count the complement:

$$(\text{total \# ways}) - (\text{\# with } \geq 11 \text{ eclairs})$$

$$\binom{25}{20} - \binom{9+6-1}{9} = \binom{25}{20} - \binom{14}{9}$$

↑
#3a

↑
sim. to #3b

9.7 # 7

$$\begin{aligned}\binom{n+3}{n+1} &= \frac{(n+3)!}{(n+1)!(n+3-(n+1))!} = \frac{(n+3) \cdot (n+2)}{(2)!} \\ &= \frac{(n+3)(n+2)}{2} \quad \text{as desired.}\end{aligned}$$

9.7 # 12 Will use $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$ repeatedly

$$\begin{aligned}S_0 \quad \binom{n+3}{r} &= \binom{n+2}{r-1} + \binom{n+2}{r} \\ &= \left(\binom{n+1}{r-2} + \binom{n+1}{r-1} \right) + \left(\binom{n+1}{r-1} + \binom{n+1}{r} \right) \\ &= \binom{n+1}{r-2} + 2 \binom{n+1}{r-1} + \binom{n+1}{r} \\ &= \binom{n}{r-3} + \binom{n}{r-2} + 2 \left(\binom{n}{r-2} + \binom{n}{r-1} \right) + \binom{n}{r-1} + \binom{n}{r} \\ &= \binom{n}{r-3} + 3 \binom{n}{r-2} + 3 \binom{n}{r-1} + 2 \binom{n}{r}\end{aligned}$$