MA 300 HW #4

Section 9.7 # 30, 38

Gri. Section 9.1 4,

9.2 9b, 10a

9.7 #30

coeff of x^7 in $(2x+3)^{10}$

$$(2x+3)^{10} = \sum_{k=0}^{10} \binom{10}{k} (2x)^k \cdot 3^{10-k}$$

$$= \sum_{k=0}^{10} \binom{10}{k} 2^k x^k \cdot 3^{10-k}$$

coeff on x^7 is $\binom{10}{7} \cdot 2^7\, 3^{10-7} = \binom{10}{7} \cdot 2^7 \cdot 3^3$

9.7 #38

Prove $\sum_{i=0}^{m} (-1)^i \binom{m}{i} 2^{m-i} = 1.$

By binomial thm here

$$1 = 1^m = (2+(-1))^m = \sum_{i=0}^{m} \binom{m}{i} \cdot 2^{m-i} \cdot (-1)^i$$ as desired.

9.1 #4a

We consider a nickel to be equivalent to 5 pennies glued together. Then taking a certain # of nickels is the same as taking a multiple of 5 pennies. So we get

$$(1+x+x^2+\dots)\quad(1+x^5+x^{10}+\dots)$$

exponents refer to # of individual pennies

exponents refer to nickels

4b

$$(1+x+x^2+\dots)(1+x^5+x^{10}+\dots)(1+x^{10}+x^{20}+\dots)$$

9.2 #6 $(x^7+x^8+\cdots)^6 = (x^7)^6(1+x+x^2+\cdots)^6 = x^{42}\cdot\left(\frac{1}{(1-x)^6}\right)$

$$= x^{42}\left(1+\binom{1+6-1}{1}x+\binom{2+6-1}{2}x^2+\cdots\right)$$

coeff on x^{50} is $\binom{8+6-1}{8}=\binom{13}{8}$.

9.2 #10a $(x^3+x^4+\cdots)^4$, we want coeff on x^{24} ← two dozen

$$(x^3)^4(1+x+\cdots)^4 = x^{12}\cdot\frac{1}{(1-x)^4}$$

$$= x^{12}\cdot\left(\underbrace{1+\binom{1+4-1}{1}x+\binom{2+4-1}{2}x^2+\cdots}\right)$$

coeff on x^{24} is like coeff on x^{12} here,

which is $\binom{12+4-1}{12}=\binom{15}{12}$