MA 300 HW #8

Section 10.1 # 2, 20, 29, 30, 39

#2

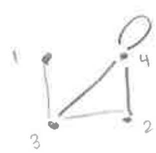
$V(G) = \{v_1, v_2, v_3, v_4\}$

$E(G) = \{e_1, \dots, e_5\}$

edge-endpt function:

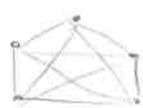
edge	endpts
e_1	$\{v_1, v_2\}$
e_2	$\{v_2, v_3\}$
e_3	$\{v_2, v_3\}$
e_4	$\{v_2, v_4\}$
e_5	$\{v_4\}$

#20 A graph with 4 verts with degrees 1, 2, 3, 4 is possible:



#29 There is a simple graph with all verts of even degree:

or



or lots of others

#30 Say G has v vertices and e edges, and each degree is at least d_{min} and at most d_{max}.

Let d_i be the degrees of each vertex, and by the handshake thm we have

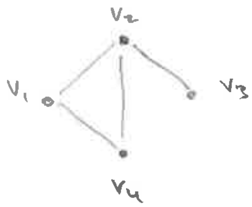
$$2e = d_1 + \dots + d_v \geq d_{min} + \dots + d_{min} = v \cdot d_{min}$$

so $e \geq \frac{1}{2} v \cdot d_{min}$.

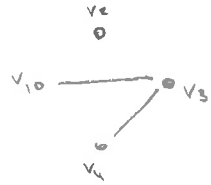
Also $2e = d_1 + \dots + d_v \leq d_{max} + \dots + d_{max} = v \cdot d_{max}$ so $e \leq \frac{1}{2} v \cdot d_{max}$

So $\frac{1}{2} d_{min}\, v \leq e \leq \frac{1}{2} d_{max}\, v$ as desired.

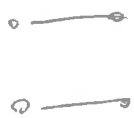
#39a



complement is



b



complement is