Name: _____

Math 1121 Exam #1

No calculators! Show all your work for everything.
For all derivatives, use the definition of the derivative. No fancy tricks!

Question 1. (15 points) Please use the definition of the derivative to find $f'(x)$ for:

$$f(x) = 5x^2 + 7x$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{5(x+h)^2+7(x+h) - (5x^2+7x)}{h} = \lim_{h\to 0} \frac{5(x^2+2xh+h^2)+7(x+h)-5x^2-7x}{h}$$

$$= \lim_{h\to 0} \frac{5x^2+10xh+5h^2+7x+7h-5x^2-7x}{h} = \lim_{h\to 0} \frac{10xh+5h^2+7h}{h}$$

$$= \lim_{h\to 0} \frac{h(10x+5h+7)}{h} = \lim_{h\to 0} 10x+5h+7 = 10x+5\cdot 0+7 = \boxed{10x+7}$$

Question 2. (10 points) Please find any discontinuities of this function:

$$f(x) = \frac{x-4}{x^2-2x-8}$$

For each discontinuity point a, find $\lim_{x\to a} f(x)$.

denom = 0 : $x^2-2x-8=0$

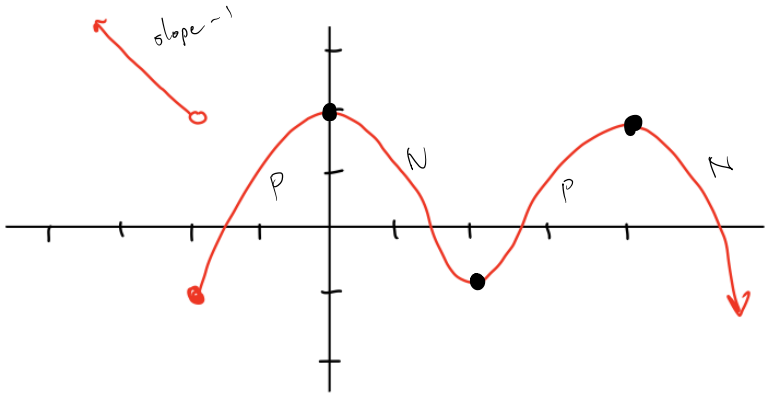
$(x-4)(x+2)=0$

$x=4, -2$ are the discontinuities

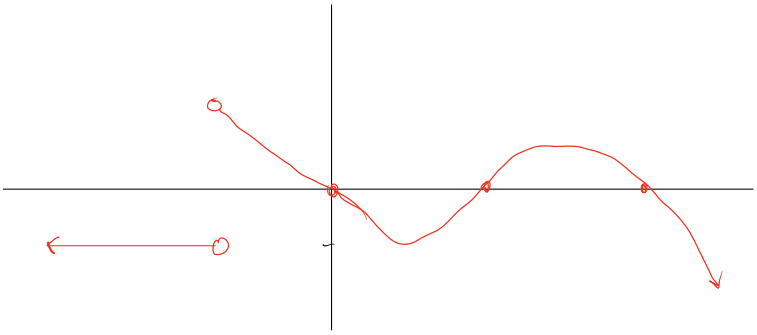
$x=4$: $\lim_{x\to 4} \frac{x-4}{x^2-2x-8} = \lim_{x\to 4} \frac{x-4}{(x-4)(x+2)} = \lim_{x\to 4} \frac{1}{x+2} = \frac{1}{4+2} = \boxed{\frac{1}{6}}$

$x=-2$: $\lim_{x\to -2} \frac{x-4}{x^2-2x-8} = \lim_{x\to 4} \frac{x-4}{(x-4)(x+2)} = \frac{-2-4}{(-2-4)\cdot 0} = \frac{-6}{0}$ $\boxed{\text{DNE}}$

Question 3. This whole page is about this picture of $f(x)$:



a) (10 points) Please sketch a graph of $f'(x)$



b) (2 points) Please give any x-values where $f(x) = 0$. (You may have to estimate.)

$x = -1.5,\ 1.5,\ 2.5,\ 5$

c) (2 points) Please give any x-values where $f'(x) = 0$. (You may have to estimate.)

$x = 0,\ 2,\ 4$

d) (3 points) Is $f'(3)$ positive or negative or zero? Say why. positive — curve is going up when $x = 3$

Please find these: (2 points each)

e) $f(-2) = -1$

f) $\lim_{x \to -2^-} f(x) = 2$

g) $\lim_{x \to -2^+} f(x) = -1$

h) $\lim_{x \to -2} f(x)$ DNE

Question 4. Please find these limits: (5 points each)

a) $\lim_{x\to 2} \dfrac{x^2+3x-10}{x-2}$ → $\dfrac{2^2+3\cdot 2-10}{2-2} = \dfrac{4+6-10}{0} = \dfrac{0}{0}$ factor

$= \lim_{x\to 2} \dfrac{(x+5)(x-2)}{x-2} = \lim_{x\to 2} x+5 = 2+5 = \boxed{7}$

b) $\lim_{x\to\infty} \dfrac{x^3-1}{x^2-1}$

degree on top is bigger, so it's DNE

c) $\lim_{x\to 1} \dfrac{2x^2-x+1}{x-3}$ $= \dfrac{2\cdot 1^2-1+1}{1-3} = \dfrac{2-1+1}{-2} = \dfrac{2}{-2} = \boxed{-1}$

d) $\lim_{x\to -1} \dfrac{(x+3)^2-4}{x+1}$ → $\dfrac{(-1+3)^2-4}{-1+1} = \dfrac{2^2-4}{-1+1} = \dfrac{0}{0}$

$= \lim_{x\to -1} \dfrac{x^2+6x+9-4}{x+1} = \lim_{x\to -1} \dfrac{x^2+6x+5}{x+1} = \lim_{x\to -1} \dfrac{(x+5)(x+1)}{x+1} = \lim_{x\to -1} x+5 = -1+5 = \boxed{4}$

Question 5. (8 points) Please find the average rate of change of:

$$f(x) = \frac{4-x}{x}$$

from $x=-2$ to $x=1$.

$$\frac{f(1)-f(-2)}{1-(-2)} = \frac{\frac{4-1}{1} - \frac{4-2}{-2}}{1+2} = \frac{3/1 - 6/-2}{3}$$

$$= \frac{3-(-3)}{3} = \frac{6}{3} = \boxed{2}$$

Question 6. (15 points) I have a slightly valuable collection of antique calculating devices (true story). Each month, the total value of my collection obeys this formula: (not a true formula)

$$f(t) = 3t + 5$$

where t is in months, and $f(t)$ is in dollars.

a) Please find $f(3)$, and say in plain language (about my collection) what this number means. Be specific about the units.

$f(3) = 3 \cdot 3 + 5 = 14$

In month 3, the total value of my collection is \$14.

b) Please find $f'(3)$, and say in plain language (about my collection) what this number means. Be specific about the units.

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{3(3+h) + 5 - (3 \cdot 3 + 5)}{h} = \lim_{h \to 0} \frac{9 + 3h + 5 - 14}{h}$$

$$= \lim_{h \to 0} \frac{3h}{h} = 3$$

In month 3, the total value of my collection is increasing at a rate of 3 \$/month.

Question 7. (7 points) Please say whether or not this function is continuous, and explain why:

$$f(x) = \begin{cases} 4x + 7 & \text{if } x < 0 \\ x + \frac{7}{x+1} & \text{if } 0 \le x \le 3 \\ x^2 + 2 & \text{if } x > 3 \end{cases}$$

near $x = 0$:

$4 \cdot 0 + 7 = 7$

$0 + \frac{7}{0+1} = 7$

it is continuous at $x = 0$

near $x = 3$:

$3 + \frac{7}{3+1} = 3 + 7/4$

$3^2 + 2 = 11$

it is not continuous at $x = 3$, since

$\lim_{x \to 3^-} f(x) = 3 + 7/4$

and $\lim_{x \to 3^+} f(x) = 11$, and these are different.