Name: _____

Math 1121 Exam #1

No calculators! Show all your work for everything.
For all derivatives, use the definition of the derivative. No fancy tricks!

Question 1. (15 points) Please use the definition of the derivative to find $f'(x)$ for:

$$f(x) = 3 - x - x^2$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3-(x+h)-(x+h)^2-(3-x-x^2)}{h} = \lim_{h\to 0} \frac{3-x-h-(x^2+2xh+h^2)-3+x+x^2}{h}$$

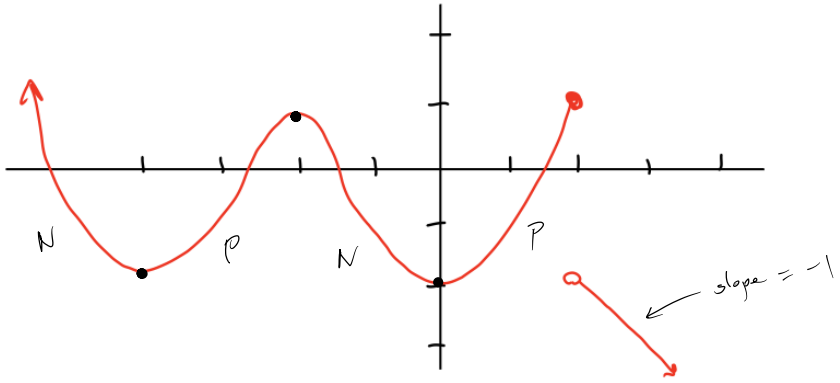
$$= \lim_{h\to 0} \frac{3-x-h-x^2-2xh-h^2-3+x+x^2}{h} = \lim_{h\to 0} \frac{-h-2xh-h^2}{h} = \lim_{h\to 0} \frac{h(-1-2x-h)}{h}$$

$$= \lim_{h\to 0} -1-2x-h = -1-2x-0 = \boxed{-1-2x}$$

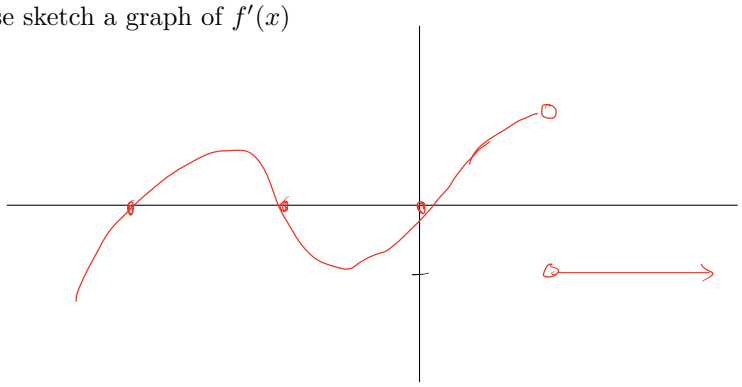
Question 2. (10 points) I make maple syrup from a tree in my back yard (true story). I also sell it! (not true) The first two jars cost \$3 each, and any additional jars cost \$2 each. Please give a formula for $p(x)$, the price to buy x jars.

$$p(x) = \begin{cases} 3x & \text{if } x \le 2 \\ 6 + 2(x-2) & \text{if } x > 2 \end{cases}$$

Question 3. This whole page is about this picture of $f(x)$:



a) (10 points) Please sketch a graph of $f'(x)$



b) (2 points) Please give intervals of x-values where $f'(x) > 0$.

$(-4,-2)$ and $(0,2)$

c) (5 points) Please find the average rate of change of $f(x)$ from $x = -2$ to $x = 0$.

$$\frac{f(0)-f(-2)}{0-(-2)} = \frac{-2-1}{2} = -3/2$$

d) (5 points) Please give the x-values for any discontinuities of $f(x)$. At each discontinuity point, find both one-sided limits. Label your answers clearly so I know what you're talking about.

$x = 2$ is the only discontinuity.

$$\lim_{x \to 2^-} f(x) = 1$$

$$\lim_{x \to 2^+} f(x) = -2$$

Question 4. (5 points each) Please find these limits:

a) $\lim_{x\to 2} \dfrac{x^2+3x-10}{x-2}$ $\to$ $\dfrac{2^2+3\cdot 2-10}{2-2} = \dfrac{4+6-10}{0} = \dfrac{0}{0}$ factor

$= \lim_{x\to 2} \dfrac{(x+5)(x-2)}{x-2} = \lim_{x\to 2} x+5 = 2+5 = \boxed{7}$

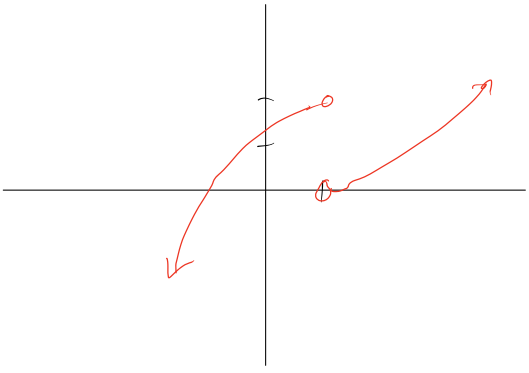
b) $\lim_{x\to\infty} \dfrac{x^3-1}{x^2-1}$ degree on top is bigger, so it's DNE

c) $\lim_{x\to 1} \dfrac{2x^2-x+1}{x-3} = \dfrac{2\cdot 1^2-1+1}{1-3} = \dfrac{2-1+1}{-2} = \dfrac{2}{-2} = \boxed{-1}$

d) $\lim_{x\to -1} \dfrac{(x+3)^2-4}{x+1}$ $\to$ $\dfrac{(-1+3)^2-4}{-1+1} = \dfrac{2^2-4}{-1+1} = \dfrac{0}{0}$

$= \lim_{x\to -1} \dfrac{x^2+6x+9-4}{x+1} = \lim_{x\to -1} \dfrac{x^2+6x+5}{x+1} = \lim_{x\to -1} \dfrac{(x+5)(x+1)}{x+1} = \lim_{x\to -1} x+5 = -1+5 = \boxed{4}$

Question 5. .



a) (6 points) Please invent and draw a picture of a graph $f(x)$ which has $\lim_{x\to 1^-} f(x) = 2$ and $\lim_{x\to 1^+} f(x) = 0$.

b) (3 points) On your picture, what is $\lim_{x\to 1} f(x)$?

DNE since the two sides have different limits

Question 6. (15 points) Each day, I sell jars of maple syrup, thereby making mad sacks of cash. Let's say that on day t, my total revenue in dollars is given by:

$$f(t) = 30t - 10.$$

a) Please find $f(3)$, and say in plain language (about my business) what this number means. Be specific about the units.

$f(3) = 30 \cdot 3 - 10 = 90 - 10 = 80$

On day 3, my total revenue is \$80.

b) Please find $f'(3)$, and say in plain language (about my business) what this number means. Be specific about the units.

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{30(3+h) - 10 - (30 \cdot 3 - 10)}{h}$$

$$= \lim_{h \to 0} \frac{90 + 30h - 10 - 80}{h} = \lim_{h \to 0} \frac{30h}{h} = 30.$$

On day 3, my revenue is increasing at a rate of \$30 per day.

Question 7. (3 points each) Consider this limit:

$$\lim_{x \to \infty} \frac{p(x)}{x^2 + 3x + 5}$$

where $p(x)$ is some polynomial.

a) For which types of polynomial $p(x)$ will the limit above equal 0?

any polynomial with degree less than 2.

b) For which types of polynomial $p(x)$ does the limit above not exist?

any polynomial with degree greater than 2

c) Is it possible for the limit above to equal 7? Either give a specific example of $p(x)$ that makes the limit 7, or explain why that would be impossible.

If is possible, like if

$p(x) = 7x^2$