Name: ____________________________

Math 1121 Exam #3

No calculators! Show all your work for everything.

$$\frac{d}{dx}e^x = e^x \qquad \int e^x\,dx = e^x + C$$

$$\frac{d}{dx}a^x = a^x \ln a \qquad \int a^x\,dx = \frac{1}{\ln a}a^x + C$$

$$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C \qquad \int a^{kx}\,dx = \frac{1}{k\ln a}a^{kx} + C$$

Question 1. (12 points) Please find the absolute extrema of $f(x) = x^3 - 3x^2$ on the interval $[-2, 1]$.

$f'(x) = 3x^2 - 6x$

$= 3x(x-2)$

$\underline{f'=0}$: $x=0$, $x=2$

x	y
0	$0^3 - 3\cdot 0^2 = 0$
outside the interval → $\cancel{2}$	
-2	$(-2)^3 - 3\cdot(-2)^2 = -8 - 3\cdot 4 = -20$
1	$1^3 - 3\cdot 1^2 = -2$

Abs max is $f(0) = 0$

Abs min is $f(-2) = -20$

Question 2. (15 points) Please find the x-values of the relative extrema for this function, and say for each one if it is a maximum or minimum.

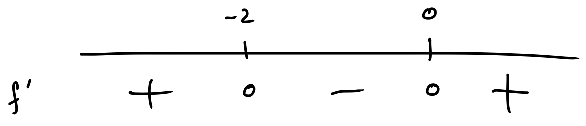
$$f(x) = x^2e^x$$

$f'(x) = x^2\cdot e^x + e^x\cdot 2x$

$= e^x(x^2+2x)$

$= e^x\cdot x\cdot(x+2)$

$f'=0$: $\cancel{e^x = 0}$ $x=0$ $x=-2$



$f'(-3) = e^{-3}\cdot(-3)(-3+2) = +\cdot - \cdot - = +$

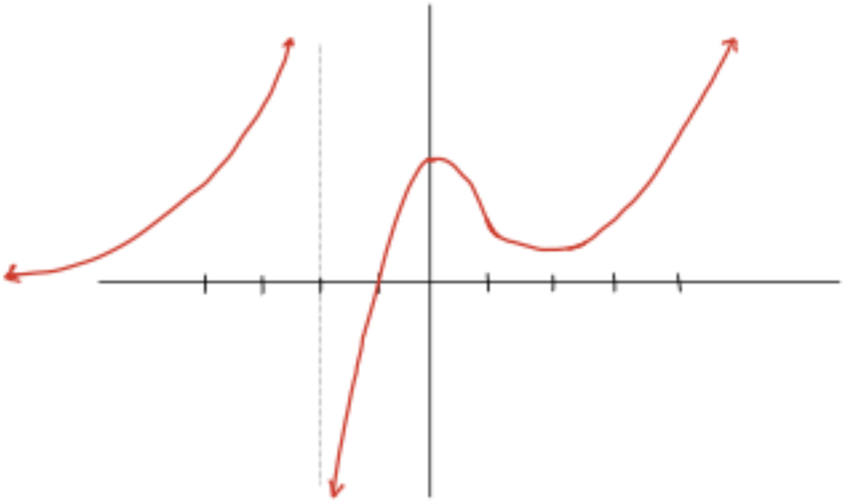
$f'(-1) = e^{-1}(-1)(-1+2) = +\cdot - \cdot + = -$

$f'(1) = e^1\cdot 1\cdot(1+2) = +\cdot+\cdot+ = +$

$x=-2$ is a rel max.

$x=1$ is a rel min.

Question 3. This whole page is about this picture of $f(x)$:



a) (6 points) Please give intervals where $f(x)$ is increasing and decreasing.

inc: $(-\infty,-2)$, $(-2,0)$, $(2,\infty)$

dec: $(0,2)$

b) (6 points) Please give intervals where $f(x)$ is concave up and concave down.

up: $(-\infty,-2)$, $(1,\infty)$

down: $(-2,1)$

c) (4 points) Please give the x-values for any relative extrema of $f(x)$, and say for each one if it is a maximum or minimum.

$x=0$ is rel max

$x=2$ is rel min

d) (4 points) Please find the x-values for any absolute extrema of $f(x)$ on the interval $[1,4]$, and say for each one if it is a maximum or minimum.

$x=4$ is abs max

$x=2$ is abs min.

Question 4. (7 points each) Please find these antiderivatives:

a) $\int 4x^3 - 8x^5 + 2\,dx$

$$4\cdot\frac{1}{4}x^4 - 8\cdot\frac{1}{6}x^6 + 2x + C$$

$$x^4 - \frac{4}{3}x^6 + 2x + C$$

b) $\int 5\sqrt{x} + 4e^{3x}\,dx$ $= \int 5x^{1/2} + 4e^{3x}\,dx$

$$= 5\cdot\frac{1}{3/2}x^{3/2} + 4\cdot\frac{1}{3}e^{3x} + C$$

c) $\int 4x^2(x^2 - \frac{1}{x^3})\,dx$ $= \int 4x^4 - 4x^{-1}\,dx$

$$= 4\cdot\frac{1}{5}x^5 - 4\ln|x| + C$$

Question 5. (12 points) Please find $f''(1)$ for $f(x) = \frac{x}{x+3}$. What does your answer tell you about the shape of the graph near $x = 1$?

$$f'(x) = \frac{(x+3)\cdot 1 - x\cdot(1)}{(x+3)^2} = \frac{x+3-x}{(x+3)^2} = \frac{3}{(x+3)^2} = 3(x+3)^{-2}$$

$$f''(x) = -6(x+3)^{-3}$$

$$f''(1) = -6(4)^{-3} = \frac{-6}{4^3}$$

negative, so it means the graph is concave down near $x=1$

(frowny shape)

Question 6. (20 points) Please sketch the graph of

$$f(x) = 3x^4 - 8x^3 + 6x^2 + 1$$

No `totes!

Show enough work so that I can tell why your picture looks the way it does.

$f'(x) = 12x^3 - 24x^2 + 12x$

$= 12x(x^2 - 2x + 1)$

$= 12x(x-1)^2$

$\underline{f' = 0}$ $\quad 12x = 0 \quad (x-1)^2 = 0$

$x = 0 \quad x = 1$

$\underline{\text{inc/dec}}$ $\quad 0 \quad 1$

f': $\quad - \quad 0 \quad + \quad 0 \quad +$

$f'(-1) = 12(-1)(\quad)^2$

$= + \cdot - \cdot +$

$f'(1/2) = 12 \cdot \frac{1}{2} \cdot (\quad)^2$

$= + + +$

$f'(2) = 12 \cdot 2 \cdot (\quad)^2$

$= + + +$

$\underline{y\text{-values}}$

$x=0$: $\quad y = f(0) = 3 \cdot 0^4 - 8 \cdot 0^3 + 6 \cdot 0^2 + 1 = 1$

$x=1$: $\quad y = f(1) = 3 \cdot 1^4 - 8 \cdot 1^3 + 6 \cdot 1^2 + 1 = 3-8+6+1 = 2$